Mark Scheme (Result)

November 2021

Pearson Edexcel GCE Further Mathematics
Advanced Level in Further Mathematics
Paper 9FM0/4C

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- M marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- A marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- B marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol √ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

Question	Scheme	Marks	AOs
1.(a)	Mass ratios: $4a^2$, $\frac{1}{2}\pi a^2$, $(4a^2+\frac{1}{2}\pi a^2)$	B1	1.2
	x: $\frac{1}{2}a$, $a+\frac{4a}{3\pi}$, $\bar{x}$ $\quad$ y: $2a$, $3a$, $\bar{y}$	B1	1.2
	Moments about OE	M1	3.1b
	$\bar{x}=\frac{(16+3\pi)a}{3(8+\pi)}$	A1	1.1b
	Moments about OA	M1	3.1b
	$\bar{y}=\frac{(16+3\pi)a}{(8+\pi)}$	A1	1.1b
		(6)	
(b)	$\tan\alpha=\frac{\bar{x}}{\bar{y}}$ and substitute for their $\bar{x}$ and $\bar{y}$	M1	3.1b
	$\tan\alpha=\frac{1}{3}$	A1	1.1b
		(2)	
			(8 marks)

Notes:

1a	B1	All correct
	B1	Distances could be measured from a parallel axis
	M1	All terms needed and must be dimensionally correct
	A1	cao (must be in terms of π and a)
	M1	All terms needed and must be dimensionally correct
	A1	cao (must be in terms of π and a)
1b	M1	Do not allow the reciprocal
	A1	cao

Question	Scheme	Marks	AOs
2(a)	$mg - kv^2 = m\dfrac{\mathrm{d}v}{\mathrm{d}t}$	M1	2.5
	Separate variables and integrate	M1	2.1
	A correct equation in any form (ignore constant or limits) e.g. $t = \dfrac{m}{k}\dfrac{1}{2\sqrt{\frac{mg}{k}}}\ln\left(\dfrac{\sqrt{\frac{mg}{k}}+v}{\sqrt{\frac{mg}{k}}-v}\right) \quad (+C)$	A1	1.1b
	$t = \dfrac{V}{2g}\ln\left(\dfrac{V+v}{V-v}\right)$ where $V^2 = \dfrac{mg}{k}$ *	A1*	2.2a
		(4)	
2(b)	$V^2 = \dfrac{mg}{k} \Rightarrow kV^2 = mg$ i.e. resistance = weight **OR** using answer to (a): As $t \to \infty$, $v \to V$ from below	B1	1.1b
	Hence V is the terminal velocity of the stone oe	B1	2.4
		(2)	
2(c)	$mg - kv^2 = mv\dfrac{\mathrm{d}v}{\mathrm{d}s}$	M1	2.5
	Separate variables and integrate	M1	2.1
	$s = -\dfrac{m}{2k}\ln\left(\dfrac{mg}{k} - v^2\right) \quad (+D)$	A1	1.1b
	$s = \dfrac{V^2}{2g}\ln\left(\dfrac{V^2}{V^2 - v^2}\right)$*	A1*	2.2a
		(4)	
			(10 marks)

Notes:		
2a	M1	Equation of motion with correct form for the acceleration
	M1	Separate the variables and integrate ('standard integral')
	A1	Correct equation in any form (ignoring constant or limits)
	A1*	Correctly obtain the printed answer including dealing with constant or limits
2b	B1	Correctly rearrange and interpret OR correctly argue and interpret
	B1	Correct statement or equivalent
2c	M1	Equation of motion with correct form for the acceleration
	M1	Separate the variables and integrate ('standard integral')

	A1	Correct equation in any form (ignoring constant or limits)
	A1*	Correctly obtain the printed answer including dealing with constant or limits

Question	Scheme	Marks	AOs
3(a)	Mass ratios: $\frac{2}{3}\pi(2a)^3$, $\frac{2}{3}\pi a^3$, $\frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3$ (8, 1, 7)	B1	1.2
	Distances: $\frac{3}{8}(2a)$, $\frac{3}{8}a$, $\bar{x}$	B1	1.2
	$\left(\frac{2}{3}\pi(2a)^3 - \frac{2}{3}\pi a^3\right)\bar{x} = \frac{2}{3}\pi(2a)^3 \times \frac{3}{8}(2a) - \frac{2}{3}\pi a^3 \times \frac{3}{8}a$	M1	3.1a
	$\bar{x} = \frac{45a}{56}$ *	A1*	2.2a
		(4)	
3(b)	Use of appropriate trig ratio e.g. $\sin\alpha = \frac{\frac{45a}{56}}{2a}$	M1	3.1a
	$\alpha = 23.7^\circ$ (3 sf)	A1	1.1b
		(2)	
			(6 marks)

Notes:		
3a	B1	Correct unsimplified (8, 1, 7)
	B1	Correct unsimplified but distances could be measured from a parallel axis
	M1	All terms needed and must be dimensionally correct
	A1*	Correct answer correctly derived
3b	M1	Must be using an appropriate trig ratio
	A1	Cao to 3SF

Question	Scheme	Marks	AOs
4(a)	$l^2 + r^2 = (2l - r)^2$, using Pythagoras	M1	1.1b
	$BR = \frac{3l}{4}$ *	A1*	1.1b
		(2)	
4(b)	Resolve vertically	M1	2.1
	$T\cos\alpha = mg$	A1	1.1b
	Overall strategy to solve problem: substitute for $\cos\alpha$ and solve for T	M1	3.1b
	$T = \frac{5mg}{4}$	A1	1.1b
		(4)	
4(c)	Equation of motion horizontally	M1	2.1
	$T + T\sin\alpha = \frac{mV^2}{r}$	A1	1.1b
	Overall strategy to solve problem: substitute for T, $\sin\alpha$ and r and solve for V	M1	3.1b
	$V = \sqrt{\frac{3gl}{2}}$	A1	1.1b
		(4)	
		(10 marks)	

Notes:		
4a	M1	Use of Pythagoras with one unknown
	A1*	Correct length
4b	M1	Allow sin/cos confusion
	A1	Correct equation
	M1	Substituting for their trig ratio and solving for T
	A1	cao
4c	M1	Correct no. of terms, dimensionally correct
	A1	Correct equation
	M1	Substitute for T, $\sin\alpha$ and r and solve for V
	A1	cao. Accept other equivalent forms

Question	Scheme	Marks	AOs
5(a)	Equation of motion along the string at the top of the circle	M1	3.1b
	$T + mg = \frac{mv^2}{a}$	A1	1.1b
	Conservation of energy	M1	3.1b
	$\frac{1}{2}mU^2 - \frac{1}{2}mv^2 = mga$	A1	1.1b
	Overall strategy to solve these equations for T and use $T = 0$	M1	3.1b
	$U = \sqrt{3ga}$	A1	1.1b
		(6)	
5(b)	Equation of motion along the string at instant string breaks	M1	3.1b
	$\frac{11mg}{2} - mg\cos\alpha = \frac{mv^2}{a}$	A1	1.1b
	Conservation of energy	M1	3.1b
	$\frac{1}{2}mv^2 - \frac{1}{2}m.4ag = mga\cos\alpha$	A1	1.1b
	Solve these equations for $\cos\alpha$ $\left(= \frac{1}{2}\right)$	M1	1.1b
	Angle turned through is 210°	A1	1.1b
		(6)	
5(c)	Find radial component of acceleration: $\frac{v^2}{a}$ $(= 5g)$	M1	2.1
	Find tangential component of acceleration: $g\sin\alpha$ $\left(= \frac{\sqrt{3}}{2}g\right)$	M1	2.1
	Square, add and square root	M1	3.1b
	$\frac{\sqrt{103}}{2}g$ or 49.7 (m s^{-2}) or 50 (m s^{-2})	A1	1.1b
		(4)	
			(16 marks)

Notes:

5a	M1	Correct number of terms
	A1	Correct equation
	M1	All terms needed and dimensionally correct
	A1	Correct equation

	M1	Solve for T and use $T = 0$ (allow $T \geq 0$)
	A1	cao
5b	M1	Correct no. of terms with mg resolved and correct acceleration component
	A1	Correct equation
	M1	All terms needed and dimensionally correct
	A1	Correct equation
	M1	Solve for $\cos\alpha$
	A1	cao
5c	M1	Uses their value of v from part (b)
	M1	Equation of motion along the tangent oe
	M1	Find the magnitude of the resultant acceleration
	A1	cao

Question	Scheme	Marks	AOs
6(a)	$mg = \dfrac{2mge}{l}$	M1	3.1a
	$e = \dfrac{1}{2}l$ so $AO = \dfrac{3l}{2}$ *	A1*	1.1b
		(2)	
6(b)	Equation of motion vertically: $mg - T = m\ddot{x}$.	M1	2.1
	$mg - \dfrac{2mg(x+e)}{l} = m\ddot{x}$.	A1	1.1b
	$-\dfrac{2g}{l}x = \ddot{x}$, so SHM with $\omega^2 = \dfrac{2g}{l}$	A1	1.1b
	Use of $\dfrac{2\pi}{\omega}$	M1	3.1a
	$2\pi\sqrt{\dfrac{l}{2g}} = \pi\sqrt{\dfrac{2l}{g}}$ *	A1*	2.2a
		(5)	
6(c)	Complete method to find h	M1	3.1a
	$mgh = \dfrac{2mg\left(\dfrac{3l}{2}\right)^2}{2l}$ **OR** $v^2 = \dfrac{2g}{l}(l^2 - (-\dfrac{1}{2}l)^2)$ and $0 = \dfrac{3gl}{2} - 2gs$	A1	1.1b
		A1	1.1b
	$h = \dfrac{9l}{4}$	A1	1.1b
		(4)	
6(d)	$-\dfrac{1}{2}l = l\cos\omega t$	M1	3.1a
	$t = \dfrac{2\pi}{3}\sqrt{\dfrac{l}{2g}}$	A1	1.1b
	$v = \sqrt{\dfrac{2g}{l}(l^2 - (-\dfrac{1}{2}l)^2}$ **OR**	M1	2.1
	$0 = \sqrt{\dfrac{3gl}{2}} - gt_1 \Rightarrow t_1 = \sqrt{\dfrac{3l}{2g}}$	M1	3.1a

		Total time = $\frac{2\pi}{3}\sqrt{\frac{l}{2g}} + \sqrt{\frac{3l}{2g}}$ oe	A1	1.1b
			(5)	
				(16 marks)

Notes:		
6a	M1	Use of Hooke's Law and $T = mg$
	A1*	Correct answer fully justified
6b	M1	All terms needed but allow a for the acceleration
	A1	Correct unsimplified equation including $\ddot{x}$
	A1	Must mention SHM
	M1	Correct method
	A1*	Correct answer correctly shown
6c	M1	Correct no. of terms, dimensionally correct in energy equation **OR** use SHM to find v^2 at unstretched position AND then use motion under gravity Correct no. of terms, dimensionally correct in both equations
	A1 A1	Equation with at most one error Correct equation **OR** correct equations
	A1	cao
6d	M1	Complete method to find time to reach unstretched position
	A1	Correct time
	M1	Complete method to find speed at unstretched position
	A1	Correct speed
	M1	Complete method to find time to rest position
	A1	Cao

Question	Scheme	Marks	AOs
7(a)	Use of an appropriate element (quarter of a circle)	M1	2.1
	$\delta A \simeq \frac{1}{2}\pi x \delta x$	A1	1.1b
	$\delta m \simeq \frac{1}{2}\pi x \delta x \times \frac{4\lambda}{\pi a^4}x^2 \ \left(=\frac{2\lambda}{a^4}x^3\delta x\right)$	A1	3.4
	$M = \int_0^a \frac{2\lambda}{a^4}x^3 \mathrm{d}x$	M1	2.1
	$M = \frac{1}{2}\lambda$	A1	1.1b
		(5)	
7(b)	Use of " $\overline{x} = \frac{1}{M}\int x \ \mathrm{d}m$ "	M1	3.4
	$= \frac{1}{M}\int_0^a \left(\frac{2\sqrt{2}x}{\pi}\right)\frac{2\lambda}{a^4}x^3 \mathrm{d}x$	A1	1.1b
	Substitute for M, integrate and sub. in limits	M1	3.4
	$\overline{x} = \frac{8\sqrt{2}a}{5\pi}$	A1	1.1b
		(4)	
			(9 marks)

Notes:		
7a	M1	Use of an appropriate element (may be implied)
	A1	Correct expression for area of element (may be implied)
	A1	Use of proportionality model to obtain mass of element (may be implied)
	M1	Integrating with correct limits
	A1	cao
7b	M1	Use the model and correct method
	A1	Correct integral
	M1	Use the model to complete the equation
	A1	cao