Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Mathematics

Advanced
Paper 1: Pure Mathematics 1

Wednesday 6 June 2018 – Morning
Time: 2 hours

Paper Reference
9MA0/01

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end

Turn over ▶

Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. Given that θ is small and is measured in radians, use the small angle approximations to find an approximate value of

$$\frac{1-\cos 4\theta}{2\theta \sin 3\theta}$$

(3)

$\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\sin\theta \approx \theta$

$\cos 4\theta \approx 1 - \frac{(4\theta)^2}{2}$ $\sin 3\theta \approx 3\theta$

$\approx 1 - \frac{16\theta^2}{2}$

$\approx 1 - 8\theta^2$

$\frac{1 - (1 - 8\theta^2)}{2\theta \cdot 3\theta}$

$\frac{1 - 1 + 8\theta^2}{6\theta^2}$

$\frac{8\theta^2}{6\theta^2}$

$\frac{8}{6}$

$\underline{\underline{\frac{4}{3}}}$

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2. A curve C has equation

$$y = x^2 - 2x - 24\sqrt{x}, \qquad x > 0$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 4$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

a/ i/ $y = x^2 - 2x - 24x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 2x - 2 - 12x^{-\frac{1}{2}}$

ii/ $\dfrac{d^2y}{dx^2} = 2 + 6x^{-\frac{3}{2}}$

b/ when $x = 4$

$\dfrac{dy}{dx} = 2(4) - 2 - 12(4)^{-\frac{1}{2}}$

$= \underline{\underline{0}}$

$\dfrac{dy}{dx} = 0 \quad \therefore$ stationary point

c/ when $x = 4$ $\quad \dfrac{d^2y}{dx^2} = 2 + 6(4)^{-\frac{3}{2}}$

$= 2.75$

$\dfrac{d^2y}{dx^2} > 0 \quad \therefore$ it is a minimum

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3.

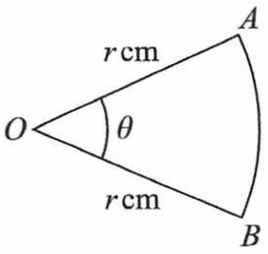

Figure 1

Figure 1 shows a sector AOB of a circle with centre O and radius r cm.

The angle AOB is θ radians.
The area of the sector AOB is 11 cm²

Given that the perimeter of the sector is 4 times the length of the arc AB, find the exact value of r.

(4)

3/

$$\frac{\theta}{2} r^2 = 11$$

$$\theta r^2 = 22 \qquad \theta = \frac{22}{r^2}$$

Arc AB $= r\theta$

$$r\theta + r + r = 4r\theta$$

$$2r + r\theta = 4r\theta$$

$$2r + r\left(\frac{22}{r^2}\right) = 4r\left(\frac{22}{r^2}\right)$$

$$2r + \frac{22}{r} = \frac{88}{r}$$

$$2r^2 + 22 = 88$$

$$2r^2 = 66$$

$$r^2 = 33$$

$$r = \sqrt{33}$$

4. The curve with equation $y = 2\ln(8 - x)$ meets the line $y = x$ at a single point, $x = \alpha$.

(a) Show that $3 < \alpha < 4$

(2)

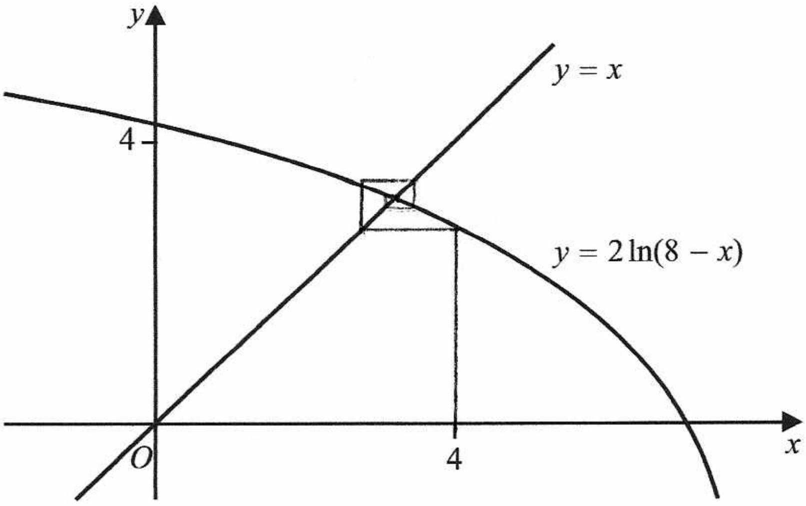

Figure 2

Figure 2 shows the graph of $y = 2\ln(8 - x)$ and the graph of $y = x$.

A student uses the iteration formula

$$x_{n+1} = 2\ln(8 - x_n), \quad n \in \mathbb{N}$$

in an attempt to find an approximation for α.

Using the graph and starting with $x_1 = 4$

(b) determine whether or not this iteration formula can be used to find an approximation for α, justifying your answer.

(2)

a/ $y = 2\ln(8-x) \qquad y = x$

$x = 2\ln(8-x)$

$0 = 2\ln(8-x) - x$

$x = 3 \qquad 2\ln(8-3) - 3 \qquad = 0.218...$

$x = 4 \qquad 2\ln(8-4) - 4 \qquad = -1.227...$

change of sign + continuous function $\therefore$ root is between 3 and 4.

Question 4 continued

b/ The Iteration formula can be used because the iterations converge on the root. (Spirals inwards)

(Total for Question 4 is 4 marks)

5. Given that

$$y = \frac{3\sin\theta}{2\sin\theta + 2\cos\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

show that

$$\frac{dy}{d\theta} = \frac{A}{1 + \sin 2\theta} \qquad -\frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

where A is a rational constant to be found.

(5)

$u = 3\sin\theta$

$v = 2\sin\theta + 2\cos\theta$

$\frac{du}{d\theta} = 3\cos\theta \qquad \frac{dv}{d\theta} = 2\cos\theta - 2\sin\theta$

$$\frac{dy}{d\theta} = \frac{3\cos\theta(2\sin\theta + 2\cos\theta) - 3\sin\theta(2\cos\theta - 2\sin\theta)}{(2\sin\theta + 2\cos\theta)^2}$$

$$= \frac{6\cos\theta\sin\theta + 6\cos^2\theta - 6\cos\theta\sin\theta + 6\sin^2\theta}{4\sin^2\theta + 8\sin\theta\cos\theta + 4\cos^2\theta}$$

$$= \frac{6\cos^2\theta + 6\sin^2\theta}{4\sin^2\theta + 4\cos^2\theta + 8\sin\theta\cos\theta}$$

$$= \frac{6(\cos^2\theta + \sin^2\theta)}{4(\sin^2\theta + \cos^2\theta) + 4(2\sin\theta\cos\theta)}$$

$\boxed{\cos^2\theta + \sin^2\theta = 1} \qquad \boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

$$= \frac{6}{4 + 4\sin 2\theta} = \frac{3}{2 + 2\sin 2\theta}$$

$$= \frac{1.5}{1 + \sin 2\theta}$$

$A = 1.5 = \frac{3}{2}$

6.

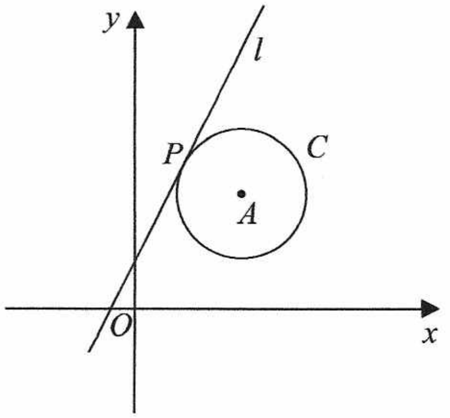

Figure 3

The circle C has centre A with coordinates (7, 5).

The line l, with equation $y = 2x + 1$, is the tangent to C at the point P, as shown in Figure 3.

(a) Show that an equation of the line PA is $2y + x = 17$

(3)

(b) Find an equation for C.

(4)

The line with equation $y = 2x + k$, $k \neq 1$ is also a tangent to C.

(c) Find the value of the constant k.

(3)

Gradient of PA $= -\frac{1}{2}$ (perpendicular)

$y = -\frac{1}{2}x + c$ (7, 5)

$5 = -\frac{1}{2}(7) + c$

$5 = -\frac{7}{2} + c$

$\frac{17}{2} = c$

$y = -\frac{1}{2}x + \frac{17}{2}$

$2y = -x + 17$

$\underline{2y + x = 17}$

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Question 6 continued

b/ point P is where $2y + x = 17$ and $y = 2x + 1$ intersect.

$$2x + 1 = -\frac{1}{2}x + \frac{17}{2}$$

$$4x + 2 = -x + 17$$

$$5x = 15$$

$$x = 3$$

$$y = 2x + 1$$
$$y = 2(3) + 1$$
$$= 7 \qquad (3, 7)$$

Length of PA $(3, 7)$ and $(7, 5)$

$$\sqrt{4^2 + 2^2}$$

$$\sqrt{20}$$

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x - 7)^2 + (y - 5)^2 = 20$$

c/ $\overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

other tangent meets at $(7, 5) + \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$(11, 3)$

$$y = 2x + k$$
$$3 = 2(11) + k \qquad k = -19$$
$$k = -19$$

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7. Given that $k \in \mathbb{Z}^+$

(a) show that $\int_k^{3k} \frac{2}{(3x-k)} \mathrm{d}x$ is independent of k,

(4)

(b) show that $\int_k^{2k} \frac{2}{(2x-k)^2} \mathrm{d}x$ is inversely proportional to k.

(3)

7a/ $\left[\frac{2}{3} \ln |3x - k| + c \right]_k^{3k}$

$\frac{2}{3} \ln(3(3k) - k) - \frac{2}{3} \ln(3(k) - k)$

$\frac{2}{3} \ln(9k - k) - \frac{2}{3} \ln(3k - k)$

$\frac{2}{3} \ln 8k - \frac{2}{3} \ln 2k$

$\frac{2}{3} (\ln 8k - \ln 2k)$

$\frac{2}{3} \ln \frac{8k}{2k}$

$\underline{\underline{\frac{2}{3} \ln 4}}$

b/ $\int_k^{2k} 2(2x - k)^{-2} \, dx$

$\left[-\frac{\cancel{2}}{\cancel{2}}(2x - k)^{-1} + c \right]_k^{2k}$

$-(2(2k) - k)^{-1} - -(2k - k)^{-1}$

$-(3k)^{-1} + k^{-1}$

$-\frac{1}{3k} + \frac{1}{k}$

$-\frac{1}{3k} + \frac{3}{3k}$

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Question 7 continued

$\frac{2}{3k}$ $\left(\alpha \frac{1}{k}\right)$

P 5 8 3 4 8 A 0 1 7 4 4

8. The depth of water, D metres, in a harbour on a particular day is modelled by the formula

$$D = 5 + 2\sin(30t)° \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

A boat enters the harbour at 6:30 am and it takes 2 hours to load its cargo.
The boat requires the depth of water to be at least 3.8 metres before it can leave the harbour.

(a) Find the depth of the water in the harbour when the boat enters the harbour.
(1)

(b) Find, to the nearest minute, the earliest time the boat can leave the harbour.
(Solutions based entirely on graphical or numerical methods are not acceptable.)
(4)

a/ $D = 5 + 2\sin(30(6.5))$

$= \underline{4.48}$ m 3sf

b/ $3.8 = 5 + 2\sin(30t)$

$-1.2 = 2\sin(30t)$

$-0.6 = \sin 30t$

$30t = \sin^{-1}(-0.6)$

$= -36.9,\ 216.9,\ 323.1,\ \cancel{576.9}$

$= -1.23,\ 7.23,\ 10.77,\ 16.52$

$10.77 = \underline{10:46}$ am

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9.

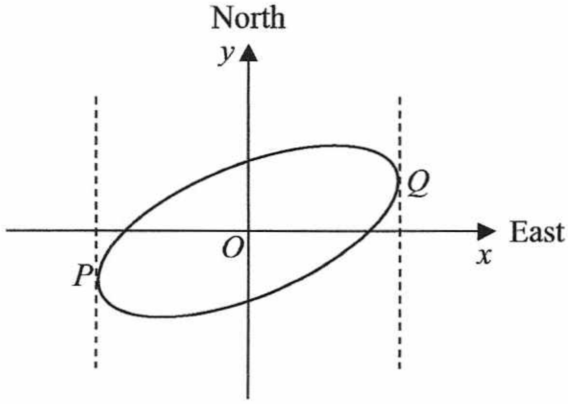

Figure 4

Figure 4 shows a sketch of the curve with equation $x^2 - 2xy + 3y^2 = 50$

(a) Show that $\dfrac{dy}{dx} = \dfrac{y - x}{3y - x}$

(4)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest west and furthest east of the origin O, as shown in Figure 4.

Using part (a),

(b) find the exact coordinates of the point P.

(5)

(c) Explain briefly how to find the coordinates of the point that is furthest north of the origin O. (You **do not** need to carry out this calculation).

(1)

$$x^2 - 2xy + 3y^2 = 50$$

$$u = -2x \quad v = y$$

$$\frac{du}{dx} = -2 \quad \frac{dv}{dx} = \frac{dy}{dx}$$

$$2x - 2x\frac{dy}{dx} - 2y + 6y\frac{dy}{dx} = 0$$

$$x - x\frac{dy}{dx} - y + 3y\frac{dy}{dx} = 0$$

$$3y\frac{dy}{dx} - x\frac{dy}{dx} = y - x$$

$$\frac{dy}{dx}(3y - x) = y - x$$

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Question 9 continued

$$\frac{dy}{dx} = \frac{y - x}{3y - x}$$

b/ P and Q are where $\frac{dx}{dy} = 0$

$$\frac{dx}{dy} = \frac{3y - x}{y - x}$$

$$\frac{3y - x}{y - x} = 0$$

$$3y - x = 0$$

$$3y = x \quad (1)$$

$$x^2 - 2xy + 3y^2 = 50 \quad (2)$$

$$(3y)^2 - 2(3y)y + 3y^2 = 50$$

$$9y^2 - 6y^2 + 3y^2 = 50$$

$$6y^2 = 50$$

$$y^2 = \frac{50}{6}$$

$$y = \pm\frac{5\sqrt{3}}{3}$$

for P $y = -\frac{5\sqrt{3}}{3}$ $\quad x = 3\left(\frac{-5\sqrt{3}}{3}\right)$

$$= -5\sqrt{3}$$

$$\left(-5\sqrt{3}, -\frac{5\sqrt{3}}{3}\right)$$

c/ where $\frac{dy}{dx} = 0$ solve $y - x = 0$ and $x^2 - 2xy + 3y^2 = 50$
pick positive solution

10. The height above ground, H metres, of a passenger on a roller coaster can be modelled by the differential equation

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

where t is the time, in seconds, from the start of the ride.

Given that the passenger is 5 m above the ground at the start of the ride,

(a) show that $H = 5e^{0.1\sin(0.25t)}$ **(5)**

(b) State the maximum height of the passenger above the ground. **(1)**

The passenger reaches the maximum height, for the second time, T seconds after the start of the ride.

(c) Find the value of T. **(2)**

$$\frac{dH}{dt} = \frac{H\cos(0.25t)}{40}$$

$$\int \frac{40}{H}\, dH = \int \cos(0.25t)\, dt$$

$$40 \ln H = 4\sin(0.25t) + C$$

when $t = 0$ $H = 5$

$$40 \ln 5 = C$$

$$40 \ln H = 4\sin(0.25t) + 40 \ln 5$$

$$\ln H = \frac{1}{10}\sin(0.25t) + \ln 5$$

$$H = e^{\frac{1}{10}\sin(0.25t) + \ln 5}$$

$$H = e^{0.1\sin(0.25t)} \cdot e^{\ln 5} \qquad \boxed{e^{\ln 5} = 5}$$

$$= 5e^{0.1\sin(0.25t)}$$

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Question 10 continued

b/ $H = 5e^{0.1\sin(0.25t)}$

Max $H = 5e^{0.1}$

$= 5.53\,m$ (3sf)

c/ Max height when $\sin(0.25t) = 1$

$0.25t = \sin^{-1}(1)$

$= \frac{\pi}{2}, \frac{5\pi}{2}$

second time

$t = 4 \cdot \frac{5\pi}{2}$

$= 10\pi$

$= 31.4$ seconds (3sf)

11. (a) Use binomial expansions to show that $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$ **(6)**

A student substitutes $x = \frac{1}{2}$ into both sides of the approximation shown in part (a) in an attempt to find an approximation to $\sqrt{6}$

(b) Give a reason why the student **should not** use $x = \frac{1}{2}$ **(1)**

(c) Substitute $x = \frac{1}{11}$ into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to $\sqrt{6}$. Give your answer as a fraction in its simplest form. **(3)**

$$(1+4x)^{\frac{1}{2}} = 1 + \tfrac{1}{2}(4x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(4x)^2$$

$$= 1 + 2x - 2x^2 \ldots$$

$$(1-x)^{-\frac{1}{2}} = 1 + (-\tfrac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-x)^2$$

$$= 1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2 \ldots$$

$$(1 + 2x - 2x^2)(1 + \tfrac{1}{2}x + \tfrac{3}{8}x^2)$$

$$1 + \tfrac{1}{2}x + 2x + \tfrac{3}{8}x^2 + x^2 - 2x^2 \ldots$$

$$1 + \tfrac{5}{2}x - \tfrac{5}{8}x^2$$

b/ The expansion of $(1+4x)^{\frac{1}{2}}$ is valid for $|x| < \frac{1}{4}$

c/ $$\sqrt{\frac{1+4(\frac{1}{11})}{1-(\frac{1}{11})}} = 1 + \tfrac{5}{2}(\tfrac{1}{11}) - \tfrac{5}{8}(\tfrac{1}{11})^2$$

$$\frac{\sqrt{6}}{2} = \frac{1183}{968}$$

$$\sqrt{6} = \frac{1183}{484}$$

12. The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,

(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

a i/ $V = Ap^t$

$32000 = Ap^4 \qquad 50000 = Ap^{11}$

$\frac{Ap^{11}}{Ap^4} = \frac{50000}{32000}$

$p^7 = \frac{25}{16}$

$p = \sqrt[7]{\frac{25}{16}}$

$= 1.0658315 58$

≈ 1.0658

ii/ $32000 = A(1.0658)^4$

$A = 24796.80$

$A \approx 24800$

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Question 12 continued

bi/ The initial value of the car (1st Jan 2001) is £24800

ii/ The value increases by 6.58% each year

c/ $100000 = 24800 \cdot 1.0658^x$

$\frac{125}{31} = 1.0658^x$

$x = \log_{1.0658}\left(\frac{125}{31}\right)$

$= 21.88$

~~22 years~~

The car will reach £100000 in the year 2022

P 5 8 3 4 8 A 0 3 5 4 4

13. Show that

$$\int_0^2 2x\sqrt{x+2}\,\mathrm{d}x = \frac{32}{15}\left(2+\sqrt{2}\right)$$

(7)

$$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$$

$$u = 2x \qquad \frac{dv}{dx} = (x+2)^{\frac{1}{2}}$$

$$\frac{du}{dx} = 2 \qquad v = \frac{2}{3}(x+2)^{\frac{3}{2}}$$

$$\left[2x\left(\frac{2}{3}(x+2)^{\frac{3}{2}}\right) - \int \frac{4}{3}(x+2)^{\frac{3}{2}}\,dx\right]_0^2$$

$$\left[\frac{4}{3}x(x+2)^{\frac{3}{2}} - \frac{8}{15}(x+2)^{\frac{5}{2}} + C\right]_0^2$$

$$\left(\frac{4}{3}(2)(2+2)^{\frac{3}{2}} - \frac{8}{15}(2+2)^{\frac{5}{2}}\right) - \left(0 - \frac{8}{15}(2)^{\frac{5}{2}}\right)$$

$$\frac{64}{15} + \frac{8}{15}(2)^{\frac{5}{2}}$$

$$\frac{64}{15} + \frac{8}{15}\cdot 4\sqrt{2}$$

$$\frac{64}{15} + \frac{32}{15}\sqrt{2}$$

$$\frac{32}{15}(2+\sqrt{2})$$

14. A curve C has parametric equations

$$x = 3 + 2\sin t, \qquad y = 4 + 2\cos 2t, \qquad 0 \leqslant t < 2\pi$$

(a) Show that all points on C satisfy $y = 6 - (x - 3)^2$

(2)

(b) (i) Sketch the curve C.

(ii) Explain briefly why C does not include all points of $y = 6 - (x - 3)^2, \quad x \in \mathbb{R}$

(3)

The line with equation $x + y = k$, where k is a constant, intersects C at two distinct points.

(c) State the range of values of k, writing your answer in set notation.

(5)

$\sin^2 t + \cos^2 t = 1$

a/ $x = 3 + 2\sin t$ $\qquad$ $y = 4 + 2\cos 2t$

$x - 3 = 2\sin t$ $\qquad$ $\boxed{\cos 2t = 2\cos^2 t - 1}$

$\frac{x-3}{2} = \sin t$ $\qquad$ $y = 4 + 2(2\cos^2 t - 1)$

$\frac{(x-3)^2}{4} = \sin^2 t$ $\qquad$ $y = 4 + 4\cos^2 t - 2$

$y = 2 + 4\cos^2 t$

$\frac{y-2}{4} = \cos^2 t$

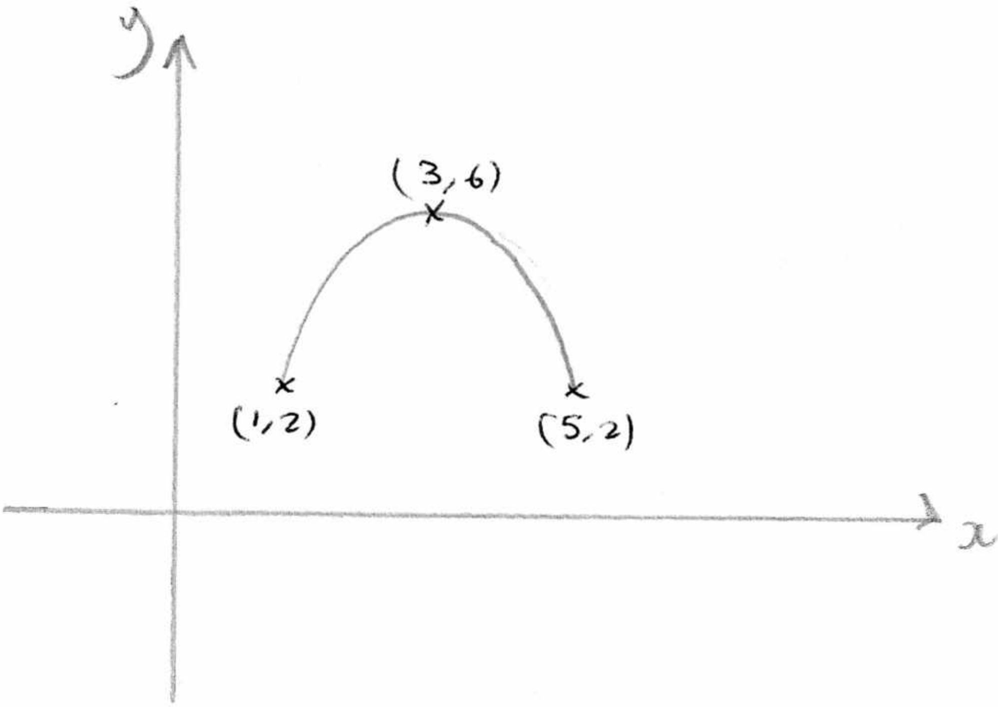
$\frac{(x-3)^2}{4} + \frac{y-2}{4} = 1$

$(x-3)^2 + y - 2 = 4$

$(x-3)^2 + y = 6$

$\underline{y = 6 - (x-3)^2}$

Question 14 continued



b/ Min $x = 3 + 2(-1)$
$= 1$

Max $x = 3 + 2(1)$
$= 5$

Maximum point $(3, 6)$

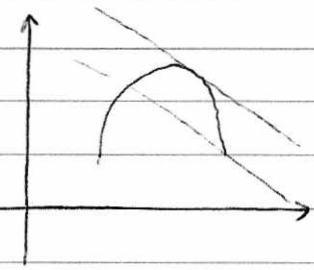
when $x = 1$ $y = 6 - (1-3)^2$
$= 2$ $(1, 2)$

$x = 5$ $y = 6 - (5-3)^2$
$= 2$ $(5, 2)$.

ii/ The domain is limited to $1 \leq x \leq 5$

Question 14 continued

c/



~~when~~ passes through $(5, 2)$

$x + y = k$

$5 + 2 = k$

$k = 7$

$y = -x + k$

tangent when $m = -1$

$y = 6 - (x-3)^2$

$\frac{dy}{dx} = -2(x-3)$

$-1 = -2(x-3)$

$\frac{1}{2} = x - 3$

$x = \frac{7}{2}$

$y = 6 - \left(\frac{7}{2} - 3\right)^2$

$= \frac{23}{4}$

$\frac{7}{2} + \frac{23}{4} = k$

$k = \frac{37}{4}$

$7 \leq k < \frac{37}{4}$

(Total for Question 14 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS