



Oxford Cambridge and RSA

Tuesday 21 June 2022 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure Mathematics

Answer **all** the questions.

1 Solve the equation $|2x - 3| = 9$. [3]

2 (a) Give full details of the single transformation that transforms the graph of $y = x^3$ to the graph of $y = x^3 - 8$. [2]

The function f is defined by $f(x) = x^3 - 8$.

(b) Find an expression for $f^{-1}(x)$. [2]

(c) State how the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are related geometrically. [1]

3 The points P and Q have coordinates $(2, -5)$ and $(3, 1)$ respectively.

Determine the equation of the circle that has PQ as a diameter. Give your answer in the form $x^2 + y^2 + ax + by + c = 0$, where a , b and c are integers. [4]

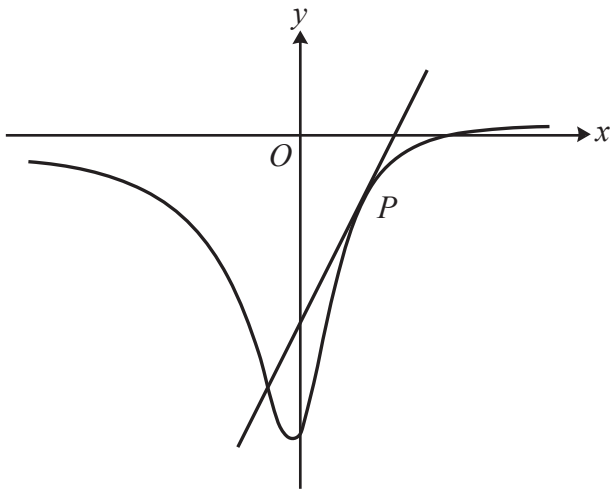
4 The positive integers x , y and z are the first, second and third terms, respectively, of an arithmetic progression with common difference -4 .

Also, x , $\frac{15}{y}$ and z are the first, second and third terms, respectively, of a geometric progression.

(a) Show that y satisfies the equation $y^4 - 16y^2 - 225 = 0$. [4]

(b) Hence determine the sum to infinity of the geometric progression. [4]

5 In this question you must show detailed reasoning.



The diagram shows the curve with equation $y = \frac{2x-3}{4x^2+1}$. The tangent to the curve at the point P has gradient 2.

(a) Show that the x -coordinate of P satisfies the equation

$$4x^3 + 3x - 3 = 0. \quad [5]$$

(b) Show by calculation that the x -coordinate of P lies between 0.5 and 1. [2]

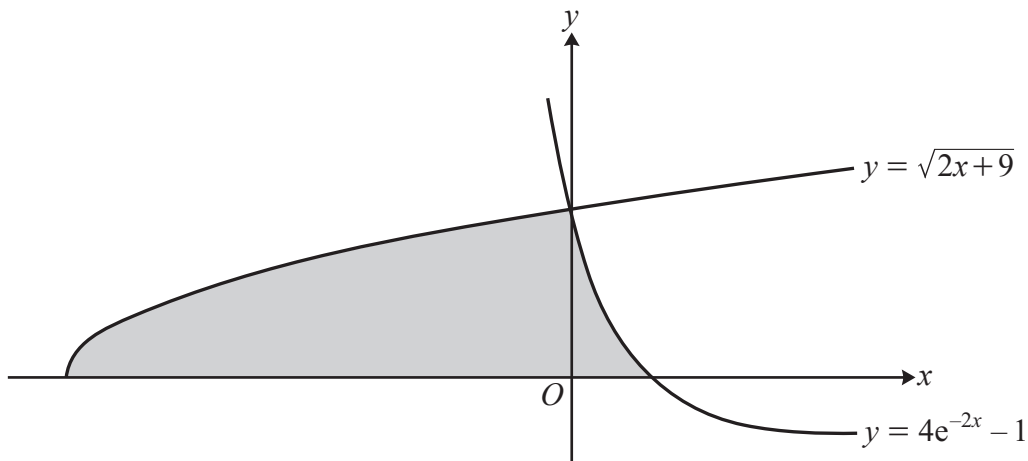
(c) Show that the iteration

$$x_{n+1} = \frac{3 - 4x_n^3}{3}$$

cannot converge to the x -coordinate of P whatever starting value is used. [2]

(d) Use the Newton-Raphson method, with initial value 0.5, to determine the coordinates of P correct to 5 decimal places. [5]

6 In this question you must show detailed reasoning.



The diagram shows the curves $y = \sqrt{2x+9}$ and $y = 4e^{-2x} - 1$ which intersect on the y -axis. The shaded region is bounded by the curves and the x -axis.

Determine the area of the shaded region, giving your answer in the form $p + q \ln 2$ where p and q are constants to be determined. [8]

7 In this question you must show detailed reasoning.

(a) Show that the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$ can be expressed in the form

$$m \tan^2 \theta - 4 \tan \theta + (m + 3) = 0. \quad [3]$$

(b) It is given that there is only one value of θ , for $0 < \theta < \pi$, satisfying the equation $m \sec \theta + 3 \cos \theta = 4 \sin \theta$.

Given also that m is a negative integer, find this value of θ , correct to 3 significant figures. [5]

Section B: Mechanics

Answer **all** the questions.

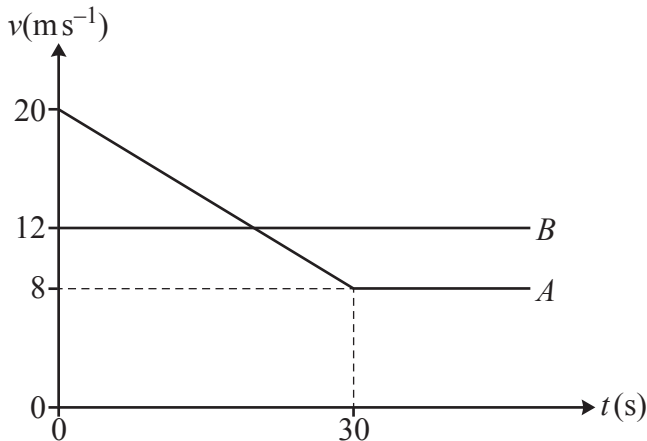
8



A child attempts to drag a sledge along horizontal ground by means of a rope attached to the sledge. The rope makes an angle of 15° with the horizontal (see diagram).

Given that the sledge remains at rest and that the frictional force acting on the sledge is 60 N , find the tension in the rope. [2]

9

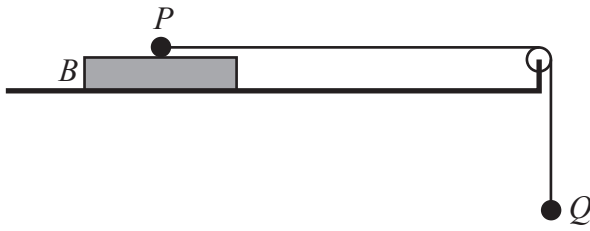


The diagram shows a velocity-time graph representing the motion of two cars A and B which are both travelling along a horizontal straight road. At time $t = 0$, car B , which is travelling with constant speed 12 m s^{-1} , is overtaken by car A which has initial speed 20 m s^{-1} .

From $t = 0$ car A travels with constant deceleration for 30 seconds. When $t = 30$ the speed of car A is 8 m s^{-1} and the car maintains this speed in its subsequent motion.

(a) Calculate the deceleration of car A . [2]

(b) Determine the value of t when B overtakes A . [4]

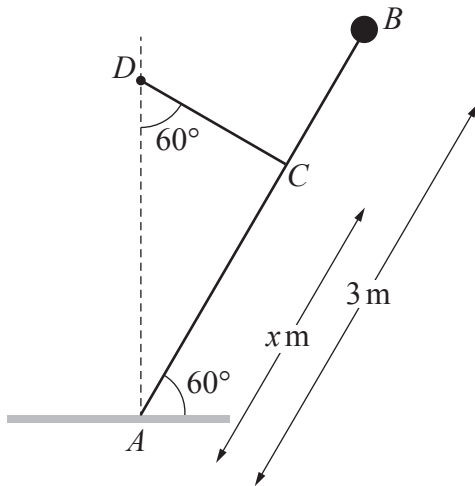


A rectangular block B is at rest on a horizontal surface. A particle P of mass 2.5 kg is placed on the upper surface of B . The particle P is attached to one end of a light inextensible string which passes over a smooth fixed pulley. A particle Q of mass 3 kg is attached to the other end of the string and hangs freely below the pulley. The part of the string between P and the pulley is horizontal (see diagram).

The particles are released from rest with the string taut. It is given that B remains in equilibrium while P moves on the upper surface of B . The tension in the string while P moves on B is 16.8 N .

- (a) Find the acceleration of Q while P and B are in contact. [2]
- (b) Determine the coefficient of friction between P and B . [3]
- (c) Given that the coefficient of friction between B and the horizontal surface is $\frac{5}{49}$, determine the least possible value for the mass of B . [3]

11



A uniform rod AB of mass 4 kg and length 3 m rests in a vertical plane with A on rough horizontal ground.

A particle of mass 1 kg is attached to the rod at B . The rod makes an angle of 60° with the horizontal and is held in limiting equilibrium by a light inextensible string CD . D is a fixed point vertically above A and CD makes an angle of 60° with the vertical. The distance AC is x m (see diagram).

(a) Find, in terms of g and x , the tension in the string. [3]

The coefficient of friction between the rod and the ground is $\frac{9\sqrt{3}}{35}$.

(b) Determine the value of x . [4]

- 12 In this question the unit vectors \mathbf{i} and \mathbf{j} are in the directions east and north respectively.

A particle P is moving on a smooth horizontal surface under the action of a single force \mathbf{F} N. At time t seconds, where $t \geq 0$, the velocity \mathbf{v} ms^{-1} of P , relative to a fixed origin O , is given by

$$\mathbf{v} = (1 - 2t)\mathbf{i} + (2t^2 + t - 13)\mathbf{j}.$$

- (a) Show that P is never stationary. [2]

- (b) Find, in terms of \mathbf{i} and \mathbf{j} , the acceleration of P at time t . [1]

The mass of P is 0.5 kg.

- (c) Determine the magnitude of \mathbf{F} when P is moving in the direction of the vector $-2\mathbf{i} + \mathbf{j}$. Give your answer correct to 3 significant figures. [5]

When $t = 1$, P is at the point with position vector $\frac{1}{6}\mathbf{j}$.

- (d) Determine the bearing of P from O at time $t = 1.5$. [5]

- 13 A small ball B moves in the plane of a fixed horizontal axis Ox , which lies on horizontal ground, and a fixed vertically upwards axis Oy . B is projected from O with a velocity whose components along Ox and Oy are $U \text{ms}^{-1}$ and $V \text{ms}^{-1}$, respectively. The units of x and y are metres.

B is modelled as a particle moving freely under gravity.

- (a) Show that the path of B has equation $2U^2y = 2UVx - gx^2$. [3]

During its motion, B just clears a vertical wall of height $\frac{1}{2}a$ m at a horizontal distance a m from O . B strikes the ground at a horizontal distance $3a$ m beyond the wall.

- (b) Determine the angle of projection of B . Give your answer in degrees correct to 3 significant figures. [5]

- (c) Given that the speed of projection of B is 54.6ms^{-1} , determine the value of a . [2]

- (d) Hence find the maximum height of B above the ground during its motion. [3]

- (e) State **one** refinement of the model, other than including air resistance, that would make it more realistic. [1]

END OF QUESTION PAPER



Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of Cambridge University Press & Assessment, which is itself a department of the University of Cambridge.