OCR
Oxford Cambridge and RSA

Tuesday 14 June 2022 – Afternoon

A Level Mathematics A

H240/02 Pure Mathematics and Statistics

Time allowed: 2 hours

You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient rule $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n\,dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

Integration by parts $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k+\tfrac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A)P(B \mid A) = P(B)P(A \mid B)$ **or** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Standard deviation

$\sqrt{\dfrac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\dfrac{\Sigma x^2}{n} - \overline{x}^2}$ **or** $\sqrt{\dfrac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\dfrac{\Sigma fx^2}{\Sigma f} - \overline{x}^2}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\overline{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p, the table gives the value of z such that $P(Z \leqslant z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 **In this question you must show detailed reasoning.**

Solve the following equations.

(a) $\frac{x}{x+1} - \frac{x-1}{x+2} = 0$ **[3]**

(b) $\frac{8}{x^6} - \frac{7}{x^3} - 1 = 0$ **[3]**

(c) $3^{x^2-7} = \frac{1}{243}$ **[2]**

2 The points A and B have position vectors $3\mathbf{i}+2\mathbf{j}$ and $4\mathbf{i}+2\mathbf{j}-5\mathbf{k}$ respectively.

(a) Find the length of AB. **[2]**

Point P has position vector $p\mathbf{i}-3\mathbf{k}$, where p is a constant. P lies on the circumference of a circle of which AB is a diameter.

(b) Find the two possible values of p. **[3]**

3 **(a)** Amaya and Ben integrated $(1+x)^2$, with respect to x, using different methods, as follows.

Amaya: $\int (1+x)^2 \, dx = \frac{(1+x)^3}{3} + c \qquad = \frac{1}{3} + x + x^2 + \frac{1}{3}x^3 + c$

Ben: $\int (1+x)^2 \, dx = \int (1+2x+x^2) \, dx \qquad = x + x^2 + \frac{1}{3}x^3 + c$

Charlie said that, because these answers are different, at least one of them must be wrong.

Explain whether you agree with Charlie's statement. **[1]**

(b) You are given that a is a constant greater than 1.

(i) Find $\int_1^a \frac{1}{(1+x)^2} \, dx$, giving your answer as a single fraction in terms of the constant a. **[3]**

(ii) You are given that the area enclosed by the curve $y = \frac{1}{(1+x)^2}$, the x-axis and the lines $x = 1$ and $x = a$ is equal to $\frac{1}{3}$.

Determine the value of a. **[2]**

(c) **In this question you must show detailed reasoning.**

Find the exact value of $\int_0^{\frac{1}{12}\pi} \frac{\cos 2x}{\sin 2x + 2} \, dx$, giving your answer in its simplest form. **[4]**

4 An artist is creating a design for a large painting. The design includes a set of steps of varying heights. In the painting the lowest step has height 20 cm and the height of each other step is 5% less than the height of the step immediately below it.

In the painting the total height of the steps is 205 cm, correct to the nearest centimetre.

Determine the number of steps in the design. **[5]**

5 **In this question you must show detailed reasoning.**

A curve has equation $y = x^3 - 3x^2 + 4x$.

(a) Show that the curve has no stationary points. **[2]**

(b) Show that the curve has exactly one point of inflection. **[2]**

6 **(a)** The diagrams show five different graphs. In each case the whole of the graph is shown.

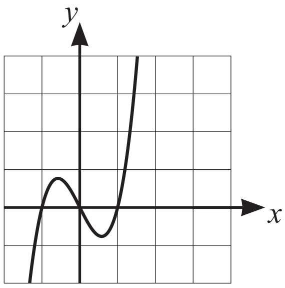


Fig. 1.1

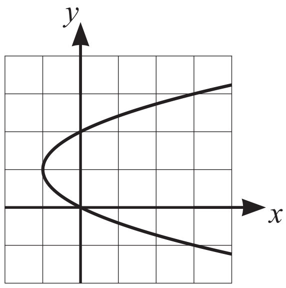


Fig. 1.2

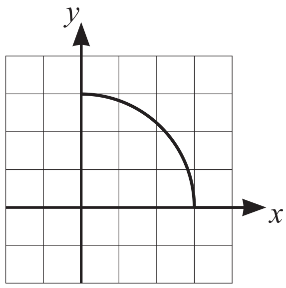


Fig. 1.3

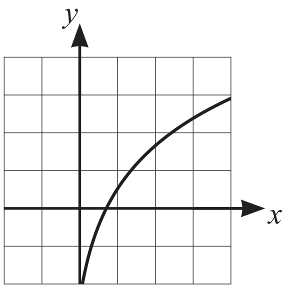


Fig. 1.4

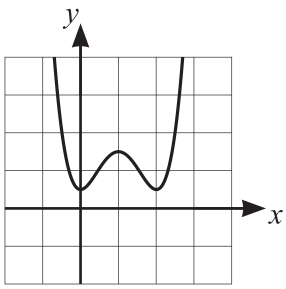


Fig. 1.5

Place ticks in the boxes in the table in the Printed Answer Booklet to indicate, for each graph, whether it represents a one-one function, a many-one function, a function that is its own inverse or it does not represent a function. There may be more than one tick in some rows or columns of the table. **[4]**

(b) A function f is defined by $\mathrm{f}(x) = \frac{1}{x}$ for the domain $\{x : 0 < x \leqslant 2\}$.

State the range of f, giving your answer in set notation. **[2]**

7 It is given that any integer can be expressed in the form $3m+r$, where m is an integer and r is 0, 1 or 2.

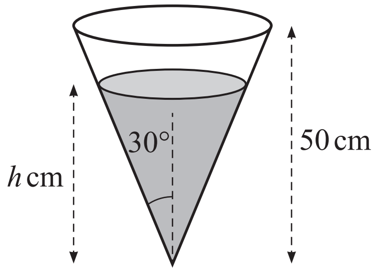
Use this fact to answer the following.

(a) By considering the different values of r, prove that the square of any integer **cannot** be expressed in the form $3n+2$, where n is an integer. **[4]**

(b) Three integers are chosen at random from the integers 1 to 99 inclusive. The three integers are not necessarily different.

By considering the different values of r, determine the probability that the sum of these three integers is divisible by 3. **[4]**

8



The diagram shows a water tank which is shaped as an inverted cone with semi-vertical angle 30° and height 50 cm. Initially the tank is full, and the depth of the water is 50 cm.

Water flows out of a small hole at the bottom of the tank. The rate at which the water flows out is modelled by $\frac{dV}{dt} = -2h$, where $V\,\text{cm}^3$ is the volume of water remaining and h cm is the depth of water in the tank t seconds after the water begins to flow out.

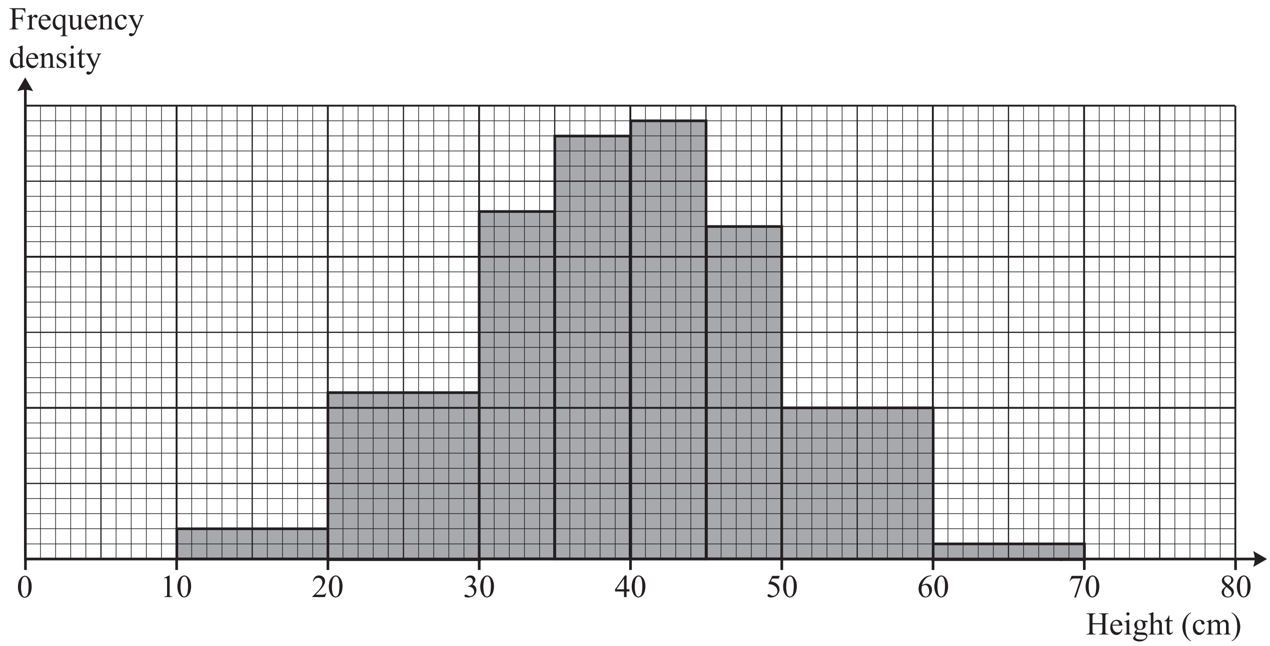
Determine the time taken for the tank to become empty.

[For a cone with base radius r and height h the volume V is given by $\frac{1}{3}\pi r^2 h$.] **[7]**

Section B: Statistics

Answer **all** the questions.

9 The heights, in centimetres, of a random sample of 150 plants of a certain variety were measured. The results are summarised in the histogram.



One of the 150 plants is chosen at random, and its height, X cm, is noted.

(a) Show that $P(20 < X < 30) = 0.147$, correct to **3** significant figures. **[2]**

Sam suggests that the distribution of X can be well modelled by the distribution $N(40, 100)$.

(b) (i) Give a brief justification for the use of the normal distribution in this context. **[1]**

(ii) Give a brief justification for the choice of the parameter values 40 and 100. **[2]**

(c) Use Sam's model to find $P(20 < X < 30)$. **[1]**

Nina suggests a different model. She uses the midpoints of the classes to calculate estimates, m and s, for the mean and standard deviation respectively, in centimetres, of the 150 heights. She then uses the distribution $N(m, s^2)$ as her model.

(d) Use Nina's model to find $P(20 < X < 30)$. **[4]**

(e) (i) Complete the table in the Printed Answer Booklet to show the probabilities obtained from Sam's model and Nina's model. **[2]**

(ii) By considering the different ranges of values of X given in the table, discuss how well the two models fit the original distribution. **[2]**

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Turn over for question 10

10 The table shows the age structure of usual residents of 18 Local Authorities (LAs) in the North West region of the UK in 2011.

Local Authority	Age 0 to 17	Age 18 to 24	Age 25 to 64	Age 65 and over
A	26.20%	9.06%	51.81%	12.92%
B	23.32%	8.99%	52.32%	15.37%
C	22.24%	8.96%	52.56%	16.23%
D	22.67%	8.10%	53.27%	15.96%
E	20.70%	7.77%	54.77%	16.76%
F	18.14%	6.51%	51.13%	24.21%
G	18.96%	14.20%	48.51%	18.33%
H	19.06%	14.79%	52.12%	14.04%
I	25.15%	9.04%	51.16%	14.65%
J	22.93%	8.81%	52.22%	16.04%
K	21.48%	13.98%	50.82%	13.73%
L	23.98%	9.20%	52.26%	14.56%
M	21.67%	11.19%	52.94%	14.19%
N	17.82%	6.01%	51.93%	24.23%
O	22.83%	7.30%	53.86%	16.01%
P	21.76%	8.28%	54.03%	15.93%
Q	21.42%	8.43%	53.90%	16.25%
R	18.61%	7.33%	49.35%	24.71%

Percentage of residents

(a) Without reference to any other columns, explain how you would use **only** the columns for the age ranges 0 to 17 and 18 to 24 to decide whether an LA might be one of the following.

(i) An LA that includes a university **[1]**

(ii) An LA that attracts young couples to live **[1]**

(iii) An LA that attracts retired people to live **[1]**

(b) Using your answers to part **(a)**, identify the following.

(i) Four LAs that might include a university **[1]**

(ii) Three LAs that might be attractive to retired people **[1]**

(c) Explain why your answer to part **(b)(ii)**, based only on the columns for the age ranges 0 to 17 and 18 to 24, may not be reliable. **[1]**

(d) The lower quartile, median and upper quartile of the percentages in the column "Age 65 and over" are 14.56%, 15.99% and 16.76% respectively.

Use this information to comment on your answers to part **(b)(ii)** and part **(c)**. **[2]**

In a magazine article, a councillor plans to describe a typical LA in the North West region. He wants to quote the average percentage of residents aged 65 or over.

(e) The mean of the percentages in the column "Age 65 and over" is 16.90%.

Use this information, and the information given in part **(d)**, to explain whether the median or the mean better represents the data in the column "Age 65 and over". **[2]**

11 In the past the masses of new-born babies in a certain country were normally distributed with mean 3300 g. Last year a publicity campaign was held to encourage pregnant women to improve their diet.

Following this campaign, it is required to test whether the mean mass of new-born babies has increased. A random sample of 200 new-born babies is chosen, and it is found that their mean mass is 3360 g. It is given that the standard deviation of the masses of new-born babies is 450 g.

Carry out the test at the 2.5% significance level. **[7]**

12 A firm claims that no more than 2% of their packets of sugar are underweight. A market researcher believes that the actual proportion is greater than 2%. In order to test the firm's claim, the researcher weighs a random sample of 600 packets and carries out a hypothesis test, at the 5% significance level, using the null hypothesis $p = 0.02$.

(a) Given that the researcher's null hypothesis is correct, determine the probability that the researcher will conclude that the firm's claim is incorrect. **[5]**

(b) The researcher finds that 18 out of the 600 packets are underweight. A colleague says

"18 out of 600 is 3%, so there is evidence that the actual proportion of underweight bags is greater than 2%."

Criticise this statement. **[1]**

Turn over for question 13

13 There are 25 students in a class.

- The number of students who study both History and English is 3.
- The number of students who study neither History nor English is 14.
- The number of students who study History but not English is three times the number who study English but not History.

(a) • Show this information on a Venn diagram.
• Determine the probability that a student selected at random studies English. **[4]**

Two different students from the class are chosen at random.

(b) Given that exactly one of the two students studies English, determine the probability that exactly one of the two students studies History. **[6]**

END OF QUESTION PAPER

