OCR
Oxford Cambridge and RSA

Wednesday 13 October 2021 – Afternoon

AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

Time allowed: 1 hour 30 minutes

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You must have:
- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- This document has **12** pages.

ADVICE
- Read each question carefully before you start your answer.

Formulae
AS Level Mathematics A (H230)

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Standard deviation

$$\sqrt{\frac{\Sigma(x-\overline{x})^2}{n}} = \sqrt{\frac{\Sigma x^2}{n} - \overline{x}^2} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\overline{x})^2}{\Sigma f}} = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \overline{x}^2}$$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np, variance of X is $np(1-p)$

Kinematics

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

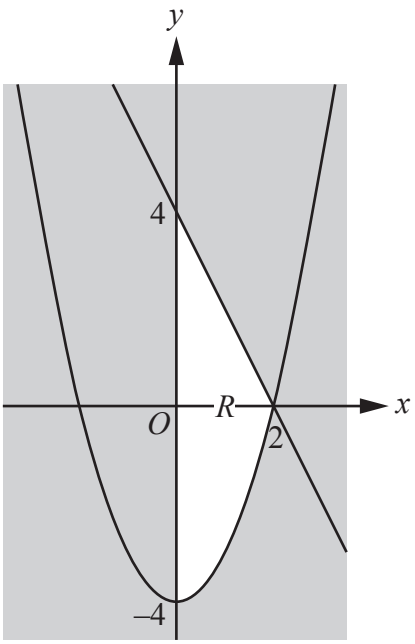
$s = vt - \frac{1}{2}at^2$

Section A: Pure Mathematics

Answer **all** the questions.

1 Given that $(x-2)$ is a factor of $2x^3+kx-4$, find the value of the constant k. **[2]**

2



The diagram shows the line $y=-2x+4$ and the curve $y=x^2-4$. The region R is the unshaded region together with its boundaries.

Write down the inequalities that define R. **[3]**

3 Sam invested in a shares scheme. The value, £V, of Sam's shares was reported t months after investment.

- Exactly 6 months after investment, the value of Sam's shares was £2375.
- Exactly 1 year after investment, the value of Sam's shares was £2825.

(a) Using a straight-line model, determine an equation for V in terms of t. **[3]**

Sam's original investment in the scheme was £1900.

(b) Explain whether or not this fact supports the use of the straight-line model in part **(a)**. **[2]**

4 The quadratic polynomial $2x^2 - 3$ is denoted by $\mathrm{f}(x)$.

Use differentiation from first principles to determine the value of $\mathrm{f}'(2)$. **[5]**

5 **(a)** Show that the equation $2\cos x \tan^2 x = 3(1 + \cos x)$ can be expressed in the form

$5\cos^2 x + 3\cos x - 2 = 0$. **[3]**

(b) **In this question you must show detailed reasoning.**

Hence solve the equation

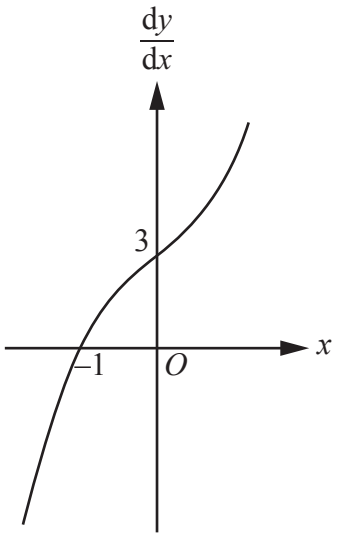
$2\cos 3\theta \tan^2 3\theta = 3(1 + \cos 3\theta)$,

giving all values of θ between 0° and 120°, correct to **1** decimal place where appropriate. **[6]**

6 A curve C has an equation which satisfies $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = 3x^2 + 2$, for all values of x.

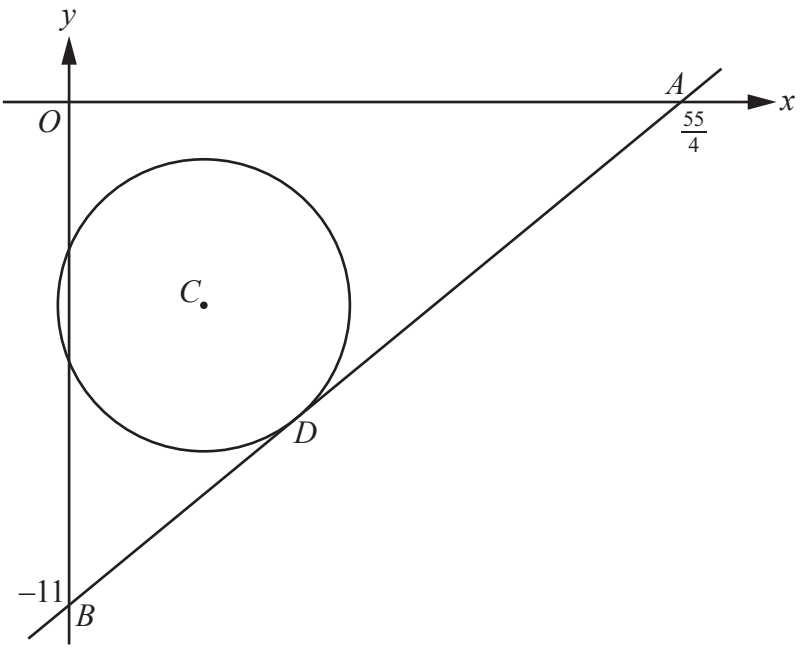
(a) It is given that C has a single stationary point. Determine the nature of this stationary point. **[1]**

The diagram shows the graph of the **gradient function** for C.



(b) Given that C passes through the point $(-1, \frac{1}{4})$, find the equation of C in the form $y = \mathrm{f}(x)$. **[5]**

7



The diagram shows the circle with equation $x^2 + y^2 - 6x + 9y + 19 = 0$ and centre C.

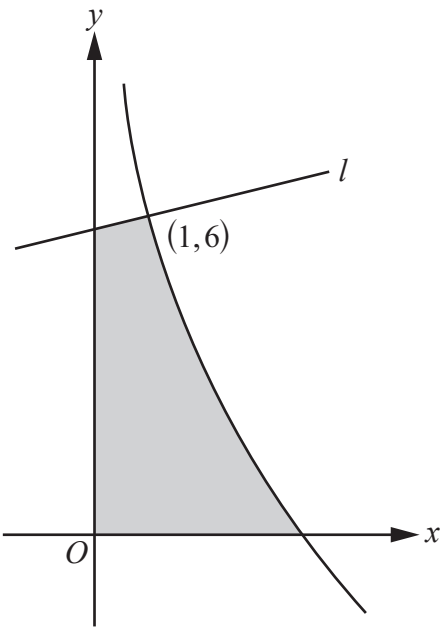
(a) Find the following.

- The coordinates of C.
- The exact radius of the circle. **[3]**

The tangent to the circle at D meets the x-axis at the point A $(\frac{55}{4}, 0)$ and the y-axis at the point B $(0, -11)$.

(b) Determine the area of triangle OBD. **[6]**

8



The diagram shows the curve $y = 1 - x + \frac{6}{\sqrt{x}}$ and the line l, which is the normal to the curve at the point (1, 6).

(a) Determine the equation of l in the form

$ax + by = c$

where a, b and c are integers whose values are to be stated. **[5]**

(b) Verify that the curve intersects the x-axis at the point where $x = 4$. **[1]**

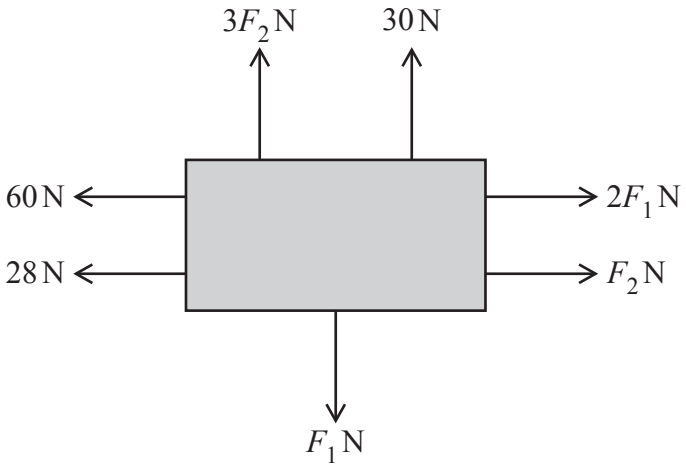
(c) **In this question you must show detailed reasoning.**

Determine the exact area of the shaded region enclosed between l, the curve, the x-axis and the y-axis. **[5]**

Section B: Mechanics

Answer **all** the questions.

9



A body remains at rest when subjected to the horizontal and vertical forces shown in the diagram.

Determine the value of F_1 and the value of F_2. **[3]**

10 A cyclist starts from rest and moves with constant acceleration along a straight horizontal road. The cyclist reaches a speed of $6\,\text{m s}^{-1}$ in 25 seconds. The cyclist then moves with constant acceleration $0.05\,\text{m s}^{-2}$ until the speed is $10\,\text{m s}^{-1}$. The cyclist then moves with constant deceleration until coming to rest. The total time for the cyclist's journey is 150 seconds.

(a) Sketch a velocity-time graph to represent the cyclist's motion. **[2]**

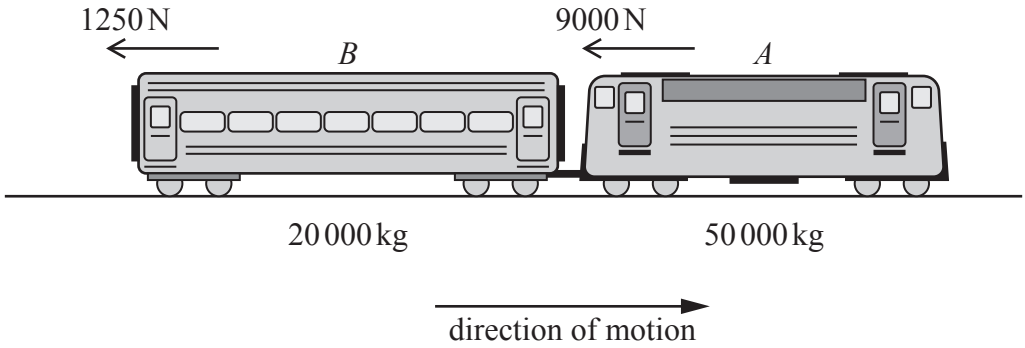
(b) Find the acceleration during the first 25 seconds of the cyclist's motion. **[1]**

The cyclist takes T seconds to decelerate from $10\,\text{m s}^{-1}$ until coming to rest.

(c) Determine the value of T. **[2]**

(d) Determine the average speed for the cyclist's journey. **[3]**

11



A train consists of an engine A of mass 50 000 kg and a carriage B of mass 20 000 kg. The engine and carriage are connected by a rigid coupling. The coupling is modelled as light and horizontal.

The resistances to motion acting on A and B are 9000 N and 1250 N respectively (see diagram).

The train passes through station P with speed $15\,\text{m s}^{-1}$ and moves along a straight horizontal track with constant acceleration $0.01\,\text{m s}^{-2}$ towards station Q. The distance between P and Q is 12.95 km.

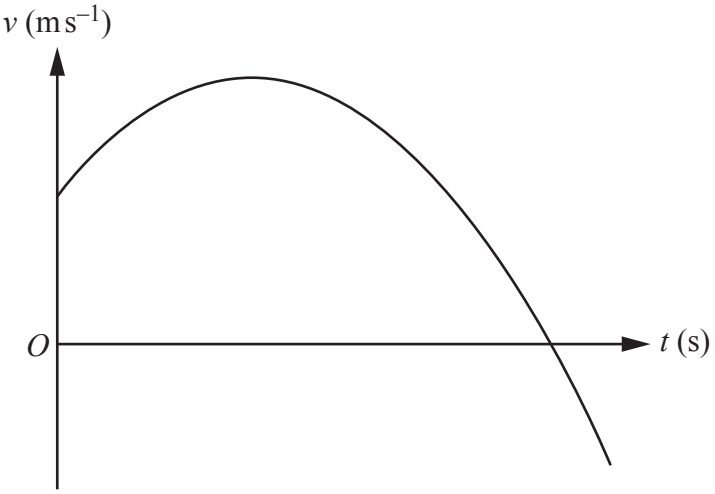
(a) Determine the time, in minutes, to travel between P and Q. **[3]**

For the train's motion between P and Q, determine the following.

(b) The driving force of the engine. **[2]**

(c) The tension in the coupling between A and B. **[2]**

12



A particle P moves in a straight line. At time t seconds, where $t \geqslant 0$, the velocity of P is $v\,\text{m s}^{-1}$. It is given that $v = -3t^2 + 24t + k$, where k is a positive constant.

The diagram shows the velocity-time graph for the motion of P.

P attains its maximum velocity at time T seconds. Given that the distance travelled by P between times $t = 1$ and $t = T$ is 297 m, determine the time when P is instantaneously at rest. **[7]**

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