



Oxford Cambridge and RSA

Monday 18 October 2021 – Afternoon

A Level Mathematics A

H240/03 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- the Printed Answer Booklet
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. You can use extra paper if you need to, but you must clearly show your candidate number, the centre number and the question numbers.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give non-exact numerical answers correct to **3** significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. When a numerical value is needed use $g = 9.8$ unless a different value is specified in the question.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **100**.
- The marks for each question are shown in brackets [].
- This document has **12** pages.

ADVICE

- Read each question carefully before you start your answer.

Formulae
A Level Mathematics A (H240)

Arithmetic series

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$\sin \theta \approx \theta$, $\cos \theta \approx 1 - \frac{1}{2}\theta^2$, $\tan \theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Standard deviation

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$, mean of X is np , variance of X is $np(1-p)$

Hypothesis test for the mean of a normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the normal distribution

If Z has a normal distribution with mean 0 and variance 1 then, for each value of p , the table gives the value of z such that $P(Z \leq z) = p$.

p	0.75	0.90	0.95	0.975	0.99	0.995	0.9975	0.999	0.9995
z	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Section A: Pure Mathematics

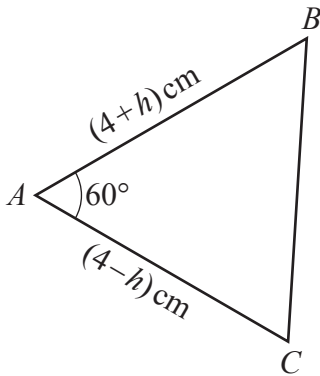
Answer **all** the questions.

- 1 Show in a sketch the region of the x - y plane within which all three of the following inequalities hold.

$$y \geq x^2, \quad x + y \leq 2, \quad x \geq 0.$$

You should indicate the region for which the inequalities hold by labelling the region R . [3]

2



The diagram shows triangle ABC in which angle A is 60° and the lengths of AB and AC are $(4+h)$ cm and $(4-h)$ cm respectively.

- (a) Show that the length of BC is p cm where

$$p^2 = 16 + 3h^2. \quad [2]$$

- (b) Hence show that, when h is small, $p \approx 4 + \lambda h^2 + \mu h^4$, where λ and μ are rational numbers whose values are to be determined. [4]

- 3 An arithmetic progression has first term 2 and common difference d , where $d \neq 0$. The first, third and thirteenth terms of this progression are also the first, second and third terms, respectively, of a geometric progression.

By determining d , show that the arithmetic progression is an increasing sequence. [5]

- 4 (a) Sketch, on a single diagram, the following graphs.
- $y = |x - 1|$
 - $y = \frac{k}{x}$, where k is a negative constant [2]
- (b) Hence explain why the equation $x|x - 1| = k$ has exactly one real root for any negative value of k . [1]
- (c) Determine the real root of the equation $x|x - 1| = -6$. [2]

- 5 A particle P moves along a straight line in such a way that at time t seconds P has velocity v m s⁻¹, where

$$v = 12 \cos t + 5 \sin t.$$

- (a) Express v in the form $R \cos(t - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 significant figures. [3]
- (b) Hence find the two smallest positive values of t for which P is moving, in either direction, with a speed of 3 m s⁻¹. [3]

- 6 The equation $6 \arcsin(2x - 1) - x^2 = 0$ has exactly one real root.

- (a) Show by calculation that the root lies between 0.5 and 0.6. [2]

In order to find the root, the iterative formula

$$x_{n+1} = p + q \sin(rx_n^2),$$

with initial value $x_0 = 0.5$, is to be used.

- (b) Determine the values of the constants p , q and r . [2]
- (c) Hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [2]

- 7 A curve C in the x - y plane has the property that the gradient of the tangent at the point $P(x, y)$ is three times the gradient of the line joining the point $(3, 2)$ to P .

(a) Express this property in the form of a differential equation. [2]

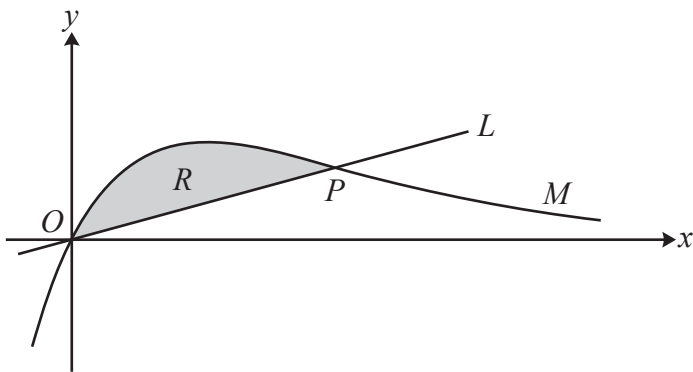
It is given that C passes through the point $(4, 3)$ and that $x > 3$ and $y > 2$ at all points on C .

(b) Determine the equation of C giving your answer in the form $y = f(x)$. [4]

The curve C may be obtained by a transformation of part of the curve $y = x^3$.

(c) Describe fully this transformation. [2]

8



The diagram shows the curve M with equation $y = xe^{-2x}$.

(a) Show that M has a point of inflection at the point P where $x = 1$. [5]

The line L passes through the origin O and the point P . The shaded region R is enclosed by the curve M and the line L .

(b) Show that the area of R is given by

$$\frac{1}{4}(a + be^{-2}),$$

where a and b are integers to be determined. [6]

Section B: Mechanics

Answer **all** the questions.

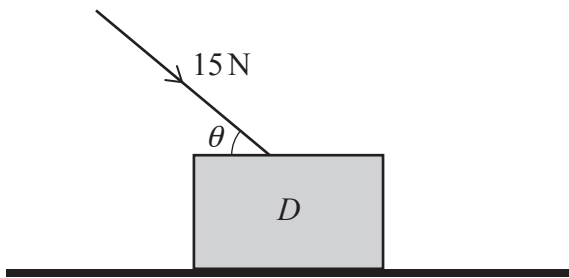
- 9 There are three checkpoints, A , B and C , in that order, on a straight horizontal road. A car travels along the road, in the direction from A to C , with constant acceleration. The car takes 20 s to travel from B to C . The speed of the car at B is 14 m s^{-1} and the speed of the car at C is 18 m s^{-1} .

(a) Find the acceleration of the car. [1]

It is given that the distance between A and B is 330 m.

(b) Determine the speed of the car at A . [2]

10



A block D of weight 50 N lies at rest in equilibrium on a fixed rough horizontal surface. A force of magnitude 15 N is applied to D at an angle θ to the horizontal (see diagram).

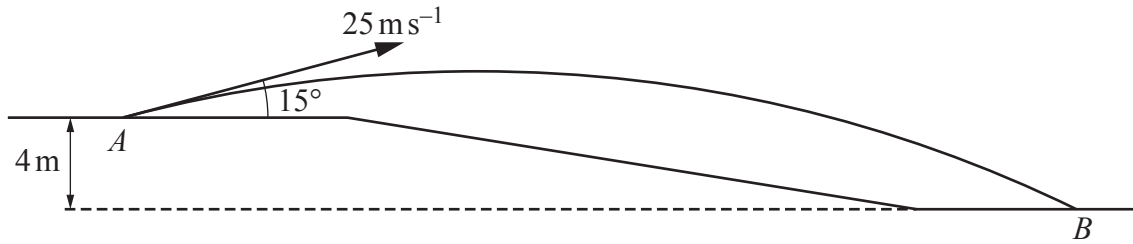
(a) Complete the diagram in the Printed Answer Booklet showing all the forces acting on D . [1]

It is given that D remains at rest and the coefficient of friction between D and the surface is 0.2.

(b) Show that

$$15 \cos \theta - 3 \sin \theta \leq 10. \quad [5]$$

11



A golfer hits a ball from a point A with a speed of 25 m s^{-1} at an angle of 15° above the horizontal. While the ball is in the air, it is modelled as a particle moving under the influence of gravity. Take the acceleration due to gravity to be 10 m s^{-2} .

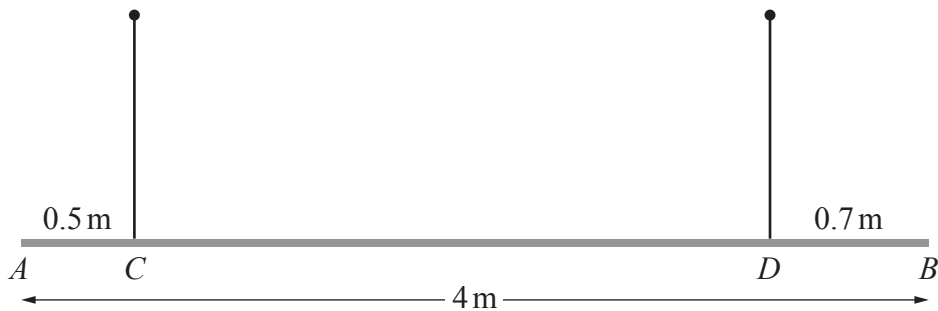
The ball first lands at a point B which is 4 m below the level of A (see diagram).

- (a) Determine the time taken for the ball to travel from A to B . [3]
- (b) Determine the horizontal distance of B from A . [2]
- (c) Determine the direction of motion of the ball 1.5 seconds after the golfer hits the ball. [4]

The horizontal distance from A to B is found to be greater than the answer to part (b).

- (d) State one factor that could account for this difference. [1]

12



A beam, AB , has length 4 m and mass 20 kg. The beam is suspended horizontally by two vertical ropes. One rope is attached to the beam at C , where $AC = 0.5 \text{ m}$. The other rope is attached to the beam at D , where $DB = 0.7 \text{ m}$ (see diagram).

The beam is modelled as a non-uniform rod and the ropes as light inextensible strings.

It is given that the tension in the rope at C is three times the tension in the rope at D .

- (a) Determine the distance of the centre of mass of the beam from A . [5]

A particle of mass $m \text{ kg}$ is now placed on the beam at a point where the magnitude of the moment of the particle's weight about C is $3.5mg \text{ N m}$. The beam remains horizontal and in equilibrium.

- (b) Determine the largest possible value of m . [2]

13 In this question the unit vectors \mathbf{i} and \mathbf{j} are in the directions east and north respectively.

At time t seconds, where $t \geq 0$, a particle P of mass 2 kg is moving on a smooth horizontal surface under the action of a constant horizontal force $(-8\mathbf{i} - 54\mathbf{j})\text{N}$ and a variable horizontal force $(4t\mathbf{i} + 6(2t - 1)^2\mathbf{j})\text{N}$.

(a) Determine the value of t when the forces acting on P are in equilibrium. [2]

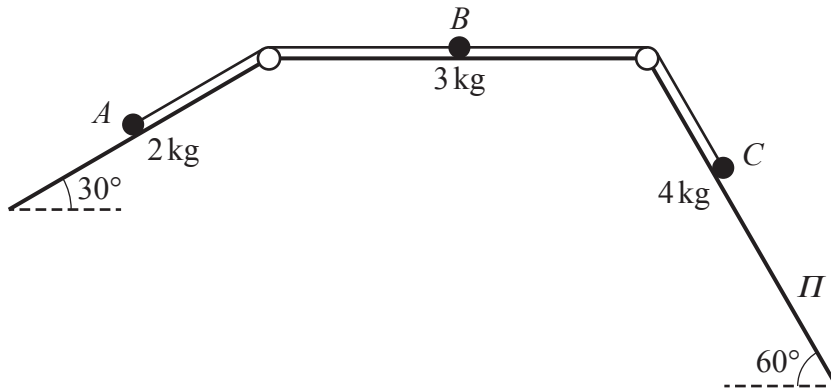
It is given that P is at rest when $t = 0$.

(b) Determine the speed of P at the instant when P is moving due north. [6]

(c) Determine the distance between the positions of P when $t = 0$ and $t = 3$. [5]

Turn over for question 14

14



One end of a light inextensible string is attached to a particle A of mass 2 kg. The other end of the string is attached to a second particle B of mass 3 kg. Particle A is in contact with a smooth plane inclined at 30° to the horizontal and particle B is in contact with a rough horizontal plane.

A second light inextensible string is attached to B . The other end of this second string is attached to a third particle C of mass 4 kg. Particle C is in contact with a smooth plane Π inclined at an angle of 60° to the horizontal.

Both strings are taut and pass over small smooth pulleys that are at the tops of the inclined planes. The parts of the strings from A to the pulley, and from C to the pulley, are parallel to lines of greatest slope of the corresponding planes (see diagram).

The coefficient of friction between B and the horizontal plane is μ . The system is released from rest and in the subsequent motion C moves down Π with acceleration $a \text{ m s}^{-2}$.

- (a) By considering an equation involving μ , a and g show that $a < \frac{1}{9}g(2\sqrt{3} - 1)$. [7]
- (b) Given that $a = \frac{1}{9}g$, determine the magnitude of the contact force between B and the horizontal plane. Give your answer correct to 3 significant figures. [4]

END OF QUESTION PAPER

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