

**Paper 4F: Further Mechanics 2 Mark Scheme**

Question	Scheme	Marks	AOs
<b>1(a)</b>	Total mass = $\int_0^{15} 10\left(1 - \frac{x}{25}\right) dx$	M1	2.1
	$= \left[10x - \frac{x^2}{5}\right]_0^{15}$	A1	1.1b
	$= 150 - \frac{225}{5} = 105 \text{ (kg) } *$	A1*	1.1b
		<b>(3)</b>	
<b>(b)</b>	Taking moments about the base: $\int_0^{15} 10x\left(1 - \frac{x}{25}\right) dx$	M1	3.4
	$= \left[5x^2 - \frac{2}{15}x^3\right]_0^{15} (= 675)$	A1	1.1b
	$\Rightarrow 105d = 675$	M1	3.4
	$d = 6.43 \text{ (m) } \quad 6\frac{3}{7} \text{ (m)}$	A1	1.1b
		<b>(4)</b>	
<b>(7 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> Use integration (usual rules)			
<b>A1:</b> Correct integration			
<b>A1*:</b> Use limits and show sufficient working to justify given answer			
<b>(b)</b>			
<b>M1:</b> Use the model to find the moment about the base (usual rules for integration)			
<b>A1:</b> Correct integration			
<b>M1:</b> Use the model to complete the moments equation Require 105 and their 675 used correctly			
<b>A1:</b> 6.43 or better			

Question	Scheme	Marks	AOs
2			
Complete overall strategy		M1	3.1b
Resolve vertically		M1	3.3
$mg + F \cos \theta = R \sin \theta$		A1	1.1b
Horizontal equation of motion		M1	3.3
$mr\omega^2 = R \cos \theta + F \sin \theta$		A1	1.1b
Use of limiting friction since maximum $\omega$		M1	3.3
Substitute for trig ratios: $\frac{3a\omega^2}{2g} = \frac{9}{2}$		M1	1.1b
Maximum $\omega = \sqrt{\frac{3g}{a}}$		A1	1.1b
<b>(8 marks)</b>			
<p><b>Notes:</b></p> <p><b>M1:</b> Overall strategy to form equation in <math>\omega</math> only e.g. consider vertical and horizontal motion and limiting friction</p> <p><b>M1:</b> Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p><b>A1:</b> Correct unsimplified equation</p> <p><b>M1:</b> Needs all 3 terms. Condone sign errors and sin/cos confusion</p> <p><b>A1:</b> Correct unsimplified equation</p> <p><b>M1:</b> Seen or implied</p> <p><b>M1:</b> Substitute to achieve equation in <math>a</math>, <math>\omega</math> and <math>g</math> only</p> <p><b>A1:</b> Or equivalent exact form</p>			

Question	Scheme	Marks	AOs												
<b>3(a)</b>	<table border="1"> <thead> <tr> <th></th> <th>mass</th> <th>c of m from <math>O</math></th> </tr> </thead> <tbody> <tr> <td>cylinder</td> <td><math>4\pi a^2 h</math></td> <td><math>\frac{h}{2}</math></td> </tr> <tr> <td>hemisphere</td> <td><math>\frac{2}{3}\pi a^3</math></td> <td><math>\frac{3}{8}a</math></td> </tr> <tr> <td><math>V</math></td> <td><math>4\pi a^2 h - \frac{2}{3}\pi a^3</math></td> <td><math>d</math></td> </tr> </tbody> </table>		mass	c of m from $O$	cylinder	$4\pi a^2 h$	$\frac{h}{2}$	hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$	$V$	$4\pi a^2 h - \frac{2}{3}\pi a^3$	$d$		
		mass	c of m from $O$												
	cylinder	$4\pi a^2 h$	$\frac{h}{2}$												
	hemisphere	$\frac{2}{3}\pi a^3$	$\frac{3}{8}a$												
	$V$	$4\pi a^2 h - \frac{2}{3}\pi a^3$	$d$												
	Mass ratios	B1	1.2												
Correct distances	B1	1.2													
Moments about a diameter through $O$	M1	2.1													
$4\pi a^2 h \times \frac{h}{2} - \frac{2}{3}\pi a^3 \times \frac{3}{8}a = 2\pi a^2 \left(2h - \frac{1}{3}a\right) \times d$	A1	1.1b													
$d = \frac{h^2 - \frac{a^2}{8}}{2h - \frac{a}{3}} = \frac{3(8h^2 - a^2)}{8(6h - a)} *$	A1*	2.2a													
	<b>(5)</b>														
<b>(b)</b>															
	$h = 5a \Rightarrow d = 2.573...a$	B1	1.1b												
	About to topple so c of m above tipping point	M1	2.2a												
	$\Rightarrow \tan \phi = \frac{2a}{5a - 2.573a}$	A1ft	1.1b												
	$\phi = 39.5^\circ$ or 0.689 rads	A1	1.1b												
	<b>(4)</b>														
<b>(9 marks)</b>															

**Question 3 notes:**

**(a)**

**B1:** Correct mass ratios

**B1:** Correct distances

**M1:** All three terms & dimensionally correct. Could use a parallel axis but final answer must be for the distance from  $O$

**A1:** Correct unsimplified equation

**A1\*:** Deduce the given answer. Their working must make it clear how they reached their answer

**(b)**

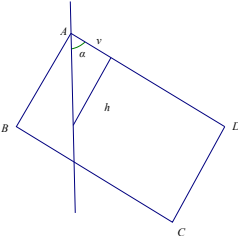
**B1:** Distance of com from base

**M1:** Condone tan the wrong way up

**A1ft:** Correct unsimplified expression for trig ratio for  $\phi$  following their  $d$

**A1:**  $39.5^\circ$  or  $0.689$  rads

Question	Scheme	Marks	AOs
<b>4(a)</b>	Equation of motion: $1800 - 2v^2 = 500a$ (when seen)	B1	2.1
	Select form for $a$ : $= 500 \frac{dv}{dt}$	M1	2.5
	$\int \frac{2}{500} dt = \int \frac{1}{900 - v^2} dv = \frac{1}{60} \int \frac{1}{30 + v} + \frac{1}{30 - v} dv$	M1	2.1
	$\frac{t}{250} = \frac{1}{60} \ln(30 + v) - \frac{1}{60} \ln(30 - v) (+C)$	A1	1.1b
	$T = \frac{25}{6} \ln\left(\frac{30+10}{30-10}\right) = \frac{25}{6} \ln 2$ *	M1 A1*	2.1 2.2a
		<b>(6)</b>	
<b>(b)</b>	Equation of motion: $500v \frac{dv}{dx} = 1800 - 2v^2$	M1	2.5
	$\int \frac{500v}{1800 - 2v^2} dv = \int 1 dx$	M1	2.1
	$-125 \ln(1800 - 2v^2) = x (+C)$	A1	1.1b
	Use boundary conditions: $x = -125 \ln 1600 + 125 \ln 1800$	M1	2.1
	$x = 125 \ln \frac{9}{8} (\text{m})$ *	A1*	2.2a
		<b>(5)</b>	
<b>(11 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>B1:</b> All three terms & dimensionally correct			
<b>M1:</b> Use of correct form for acceleration to give equation in $v, t$ only			
<b>M1:</b> Separate variables and integrate			
<b>A1:</b> Condone missing $C$			
<b>M1:</b> Use boundary conditions correctly			
<b>A1*:</b> Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			
<b>(b)</b>			
<b>M1:</b> Correct form of acceleration in the equation of motion to give equation in $v, x$ only			
<b>M1:</b> Separate variables and integrate			
<b>A1:</b> Condone missing $C$			
<b>M1:</b> Extract and use boundary conditions			
<b>A1*:</b> Show sufficient working to justify given answer and a 'statement' that the required result has been achieved			

Question	Scheme	Marks	AOs												
<b>5(a)</b>	<table border="1"> <thead> <tr> <th></th> <th>Mass</th> <th>From <math>AD</math></th> </tr> </thead> <tbody> <tr> <td>Rectangle</td> <td><math>8a^2</math></td> <td><math>a</math></td> </tr> <tr> <td>Semicircle</td> <td><math>\frac{1}{2}\pi a^2</math></td> <td><math>\frac{4a}{3\pi}</math></td> </tr> <tr> <td>Sign</td> <td><math>a^2\left(8 - \frac{\pi}{2}\right)</math></td> <td><math>h</math></td> </tr> </tbody> </table>		Mass	From $AD$	Rectangle	$8a^2$	$a$	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$	Sign	$a^2\left(8 - \frac{\pi}{2}\right)$	$h$		
		Mass	From $AD$												
	Rectangle	$8a^2$	$a$												
	Semicircle	$\frac{1}{2}\pi a^2$	$\frac{4a}{3\pi}$												
	Sign	$a^2\left(8 - \frac{\pi}{2}\right)$	$h$												
	Mass ratios		B1	1.2											
Moments about $AD$		M1	2.1												
$a^2\left(8 - \frac{\pi}{2}\right)h = 8a^2 \times a - \frac{1}{2}\pi a^2 \times \frac{4a}{3\pi} \left(= 8a^3 - \frac{2}{3}a^3 = \frac{22}{3}a^3\right)$		A1	1.1b												
$\Rightarrow h = \frac{22}{3}a \div \left(8 - \frac{\pi}{2}\right) = \frac{44a}{3(16 - \pi)}$ *		A1*	2.2a												
		<b>(4)</b>													
<b>(b)</b>	Moments about $A$ $2aT = \frac{44a}{3(16 - \pi)}W$	M1	3.1b												
	$T = \frac{hW}{2a} = \frac{22W}{3(16 - \pi)}$	A1	1.1b												
		<b>(2)</b>													
<b>(c)</b>															
	Take moments about $AB$ to find distance of com from $AB$	M1	3.1b												
	$8a^2 \times 2a - \frac{1}{2}\pi a^2 \times d = \left(8 - \frac{1}{2}\pi\right)a^2 \times v$	A1	1.1b												
	$v = \frac{32a - \pi d}{16 - \pi}$	A1	1.1b												
	Correct trig for the given angle	M1	3.1b												
	$\tan \alpha = \frac{11}{18} = \frac{h}{v} = \frac{44a}{3(32a - \pi d)}$	A1ft	1.1b												
	$(24a = 32a - \pi d, \quad 8a = \pi d) \quad d = \frac{8a}{\pi}$	A1	1.1b												
		<b>(6)</b>													
<b>(12 marks)</b>															

**Question 5 notes:**

**(a)**

**B1:** Correct mass ratios

**M1:** Need all three terms, must be dimensionally correct

**A1:** Correct unsimplified equation

**A1\*:** Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved

**(b)**

**M1:** Could also take moments about B **or** about the c.o.m. and use

**A1:** cso

**(c)**

**M1:** All terms and dimensionally correct

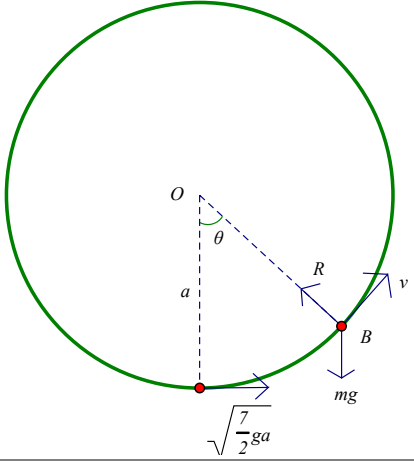
**A1:** Correct unsimplified equation

**A1:** Or equivalent

**M1:** Condone tan the wrong way up

**A1:** Equation in a and d; follow through on their v

**A1:** cao

Question	Scheme	Marks	AOs
<b>6(a)</b>			
	Conservation of energy	M1	2.1
	$\frac{1}{2}mv^2 + mga(1 - \cos \theta) = \frac{1}{2}m\left(\frac{7}{2}ga\right)$	A1	1.1b
	$v^2 = ga\left(\frac{3}{2} + 2\cos \theta\right)^*$	A1*	2.2a
	<b>(3)</b>		
<b>(b)</b>	Resolve parallel to $OB$ and use $\frac{mv^2}{a}$	M1	3.1b
	$R - mg \cos \theta = \frac{mv^2}{a}$	A1	1.1b
	Use $R = 0$ $g \cos \theta = -\frac{v^2}{a}$	M1	3.1b
	Solve for $\theta \Rightarrow g \cos \theta = -g\left(\frac{3}{2} + 2\cos \theta\right)$	M1	1.1b
	$\theta = 120^\circ$	A1	1.1b
<b>(5)</b>			
<b>(c)</b>	Any appropriate comment e.g. the hoop is unlikely to be smooth	B1	3.5b
	<b>(1)</b>		



Question	Scheme	Marks	AOs
<b>6(d)</b>	At rest $\Rightarrow v = 0$	M1	3.1b
	$\Rightarrow \cos \theta = -\frac{3}{4}$	A1	1.1b
	Acceleration is tangential	M1	3.1b
	Magnitude $ g \cos(\theta - 90)  = 6.48 \text{ m s}^{-2}$ or $\frac{\sqrt{7}}{4} g$	A1	1.1b
	At $\left(\cos^{-1}\left(-\frac{3}{4}\right) - 90 = \right) 48.6^\circ$ to the downward vertical	A1	1.1b
		<b>(5)</b>	
<b>(14 marks)</b>			
<b>Question 6 notes:</b>			
<b>(a)</b>			
<b>M1:</b> All terms required. Must be dimensionally correct			
<b>A1:</b> Correct unsimplified equation			
<b>A1*:</b> Show sufficient working to justify the given answer and a 'statement' that the required result has been achieved			
<b>(b)</b>			
<b>M1:</b> Resolve parallel to $OB$			
<b>A1:</b> Correct equation			
<b>M1:</b> Use $R=0$ seen or implied			
<b>M1:</b> Solve for $\theta$			
<b>A1:</b> Accept $\frac{2\pi}{3}$			
<b>(c)</b>			
<b>B1:</b> Any appropriate comment e.g. - hoop may not be smooth; - air resistance could affect the motion			
<b>(d)</b>			
<b>M1:</b> $v = 0$ seen or implied			
<b>A1:</b> Correct equation in $\theta$			
<b>M1:</b> Correct direction for acceleration			
<b>A1:</b> Accept 6.48, 6.5 or exact in $g$			
<b>A1:</b> Accept 0.848 (radians)			

Question	Scheme	Marks	AOs
<b>7(a)</b>			
	$T_A = \frac{20e}{2}, T_B = \frac{50(2-e)}{2} e$	M1	3.1a
	In equilibrium $T_A = T_B$ , $10e = 25(2-e)$	M1	3.1a
	$(35e = 50), e = \frac{10}{7}$	A1	1.1b
	Equation of motion for $P$ when distance $x$ from equilibrium position towards $B$ :	M1	3.1a
	$3.5\ddot{x} = T_B - T_A = \frac{50(2-e-x)}{2} - \frac{20(e+x)}{2}$	A1 A1	1.1b 1.1b
	$= \frac{50\left(\frac{4}{7}-x\right)}{2} - \frac{20\left(\frac{10}{7}+x\right)}{2}$		
	$\Rightarrow 3.5\ddot{x} = -35x, \ddot{x} = -10x$ <p>and hence SHM about the equilibrium position</p>	A1	3.2a
	<b>(7)</b>		
<b>(b)</b>	Amplitude $= 2 - \frac{10}{7} = \frac{4}{7}$	B1 ft	2.2a
	Use of max speed $= a \omega$	M1	1.1b
	$= \frac{4}{7} \sqrt{10} = 1.81 \text{ (m s}^{-1}\text{)}$	A1 ft	1.1b
		<b>(3)</b>	

Question	Scheme	Marks	AOs
7(c)	Nearer to $A$ than to $B$ : $x < -\frac{3}{7}$	B1	3.1a
	Solve for $\sqrt{10}t$ : $\cos \sqrt{10}t = -\frac{3}{4}$ , $\sqrt{10}t = 2.418\dots\dots\dots$	M1	3.1a
	Length of time: $\frac{2}{\sqrt{10}}(\pi - 2.418\dots)$	M1	1.1b
	0.457 (seconds)	A1	1.1b
	Alternative: $\frac{3.864 - 2.419}{\sqrt{10}} = 0.457$		
	Alternative: $x = \frac{4}{7} \sin \sqrt{10}t = \frac{3}{7} \Rightarrow \sqrt{10}t = 0.8481$ or $\sqrt{10}t = 2.29353$ $t_1 = 0.2682$ , $t_2 = 0.72527$ $\Rightarrow$ time = 0.457 (seconds)		
		(4)	
<b>(14 marks)</b>			
<b>Notes:</b>			
<b>(a)</b>			
<b>M1:</b> Use of $T = \frac{\lambda x}{a}$			
<b>M1:</b> Dependent on the preceding M1. Equate their tensions			
<b>A1:</b> cao			
<b>M1:</b> Condone sign error			
<b>A1:</b> Correct unsimplified equation in $e$ and $x$ A1A1 Equation with one error A1A0			
<b>A1:</b> Full working to justify conclusion that it is SHM about the equilibrium position			
<b>(b)</b>			
<b>B1ft:</b> Seen or implied. Follow their $e$			
<b>M1:</b> Correct method for max. speed			
<b>A1ft:</b> 1.81 or better. Follow their $a, \omega$			
<b>(c)</b>			
<b>B1:</b> Seen or implied			
<b>M1:</b> Use of $x = a \cos \omega t$			
<b>M1:</b> Correct strategy for the required interval			
<b>A1:</b> 0.457 or better			

