Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 14 October 2020

Afternoon (Time: 2 hours) | Paper Reference **9MA0/02**

Mathematics

Advanced
Paper 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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1 The table below shows corresponding values of x and y for $y = \sqrt{\frac{x}{1+x}}$

The values of y are given to 4 significant figures.

x	0.5	1	1.5	2	2.5
y	0.5774	0.7071	0.7746	0.8165	0.8452

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}}\, dx$$

giving your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}}\, dx = 4.535 \text{ to 4 significant figures}$$

(c) comment on the accuracy of your answer to part (b).

(1)

a/ $0.5\left(\frac{0.5774}{2} + 0.7071 + 0.7746 + 0.8165 + \frac{0.8452}{2}\right)$

$= 1.50$ 3sf

b/ $\sqrt{\frac{9x}{1+x}}$

$\sqrt{9}\sqrt{\frac{x}{1+x}}$

$3\sqrt{\frac{x}{1+x}}$

$3(1.50) = 4.50$ [3×Ans = 4.51]

c/ 4.535 is close to 4.50. The estimate is quite accurate.

Question 1 continued

(Total for Question 1 is 5 marks)

2. Relative to a fixed origin, points P, Q and R have position vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ respectively.

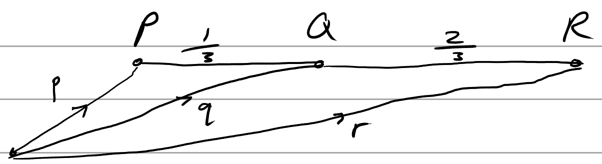
Given that

- P, Q and R lie on a straight line
- Q lies one third of the way from P to R

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)



$\overrightarrow{PQ} = -p + q$ $\qquad \overrightarrow{PR} = -p + r$

$\overrightarrow{PQ} = \frac{1}{3}(-p + r)$

$-p + q = \frac{1}{3}(-p + r)$

$q = -\frac{1}{3}p + \frac{1}{3}r + p$

$= \frac{2}{3}p + \frac{1}{3}r$

$= \frac{1}{3}(2p + r)$

$= \frac{1}{3}(r + 2p)$

Question 2 continued

(Total for Question 2 is 3 marks)

3. (a) Given that

$$2\log(4-x) = \log(x+8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2\log(4-x) = \log(x+8)$$

giving a reason for your answer.

(2)

a/ $2\log(4-x) = \log(x+8)$

$\log(4-x)^2 = \log(x+8)$

$(4-x)^2 = x+8$

$(4-x)(4-x) = x+8$

$16-4x-4x+x^2 = x+8$

$x^2-8x+16 = x+8$

$x^2-9x+8 = 0$

b/ $(x-8)(x-1) = 0$

$x=8$ $x=1$

ii/ 8 cannot be a solution

$\log(4-8) = \log -4$

$= \log_{10} -4$

$10^y \neq$ A negative number

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Question 3 continued

(Total for Question 3 is 5 marks)

4. In the binomial expansion of

$$(a + 2x)^7 \qquad \text{where } a \text{ is a constant}$$

the coefficient of x^4 is 15 120

Find the value of a.

(3)

$^7C_4 = 35$

$35a^3(2x)^4$

$560a^3x^4 = 15120x^4$

$560a^3 = 15120$

$a^3 = 27$

$a = 3$

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Question 4 continued

(Total for Question 4 is 3 marks)

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P.

Find, using algebra, the exact x coordinate of P.

(4)

$3 \times 2^x = 15 - 2^{x+1}$

$3(2^x) = 15 - 2(2^x)$

$5(2^x) = 15$

$2^x = 3$

$x = \log_2 3$

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Question 5 continued

(Total for Question 5 is 4 marks)

6\. (a) Given that

$$\frac{x^2+8x-3}{x+2} \equiv Ax + B + \frac{C}{x+2} \qquad x \in \mathbb{R} \;\; x \neq -2$$

find the values of the constants A, B and C

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2+8x-3}{x+2}\,dx$$

giving your answer in the form $a + b\ln 2$ where a and b are integers to be found.

(4)

$$x + 6$$

$$x+2 \overline{)\, x^2 + 8x - 3}$$

$$x^2 + 2x$$

$$6x - 3$$

$$6x + 12$$

$$-15$$

$$\frac{x^2+8x-3}{x+2} = x + 6 - \frac{15}{x+2}$$

$A = 1 \quad B = 6 \quad C = -15$

b/ $\int_0^6 x + 6 - \frac{15}{x+2} \, dx$

$$\left[\frac{1}{2}x^2 + 6x - 15\ln|x+2|\right]_0^6$$

$$\left(\frac{1}{2}(6)^2 + 6(6) - 15\ln 8\right) - \left(-15\ln 2\right)$$

$18 + 36 - 15\ln 8 + 15\ln 2$

$54 - 15\ln 2^3 + 15\ln 2$

$54 - 45\ln 2 + 15\ln 2$

$54 - 30\ln 2$

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Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7.

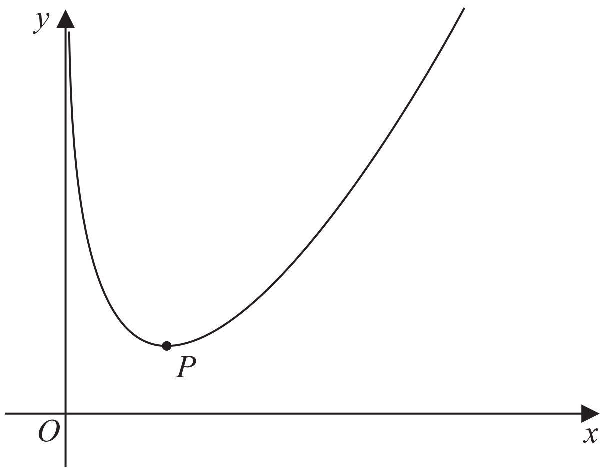

Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4\ln x \qquad x > 0$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

(4)

The point P, shown in Figure 1, is the minimum turning point on C.

(b) Show that the x coordinate of P is a solution of

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

(3)

(c) Use the iteration formula

$$x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}} \qquad \text{with } x_1 = 2$$

to find (i) the value of x_2 to 5 decimal places,

(ii) the x coordinate of P to 5 decimal places.

(3)

$$y = \frac{4x^2}{2x^{\frac{1}{2}}} + \frac{x}{2x^{\frac{1}{2}}} - 4\ln x$$

Question 7 continued

$$y = 2x^{\frac{3}{2}} + \frac{1}{2}x^{\frac{1}{2}} - 4\ln x$$

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{1}{4}x^{-\frac{1}{2}} - \frac{4}{x}$$

$$= 3\sqrt{x} + \frac{1}{4\sqrt{x}} - \frac{4}{x}$$

$$= \frac{4x\sqrt{x}(3\sqrt{x})}{4x\sqrt{x}} + \frac{x}{4x\sqrt{x}} - \frac{4(4\sqrt{x})}{4x\sqrt{x}}$$

$$= \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

b/ Turning point is where $\frac{dy}{dx} = 0$

$$\frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}} = 0$$

$$12x^2 + x - 16\sqrt{x} = 0$$

$$12x^{\frac{3}{2}} + \sqrt{x} - 16 = 0$$

$$12x^{\frac{3}{2}} = 16 - \sqrt{x}$$

$$x^{\frac{3}{2}} = \frac{16}{12} - \frac{\sqrt{x}}{12}$$

$$x = \left(\frac{4}{3} - \frac{\sqrt{x}}{12}\right)^{\frac{2}{3}}$$

c/ $x_{n+1} = \left(\frac{4}{3} - \frac{\sqrt{x_n}}{12}\right)^{\frac{2}{3}}$ $\qquad x_1 = 2$

$$x_2 = \left(\frac{4}{3} - \frac{\sqrt{2}}{12}\right)^{\frac{2}{3}}$$

$= \underline{1.13894}$

ii) $x_3 = 1.15693$ $\qquad x_5 = 1.15651$

$x_4 = 1.15649$ $\qquad x_6 = \underline{1.15650}$

Turn over ▶

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8. A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = 6x^2 + ax - 23$ where a is a constant
- the y intercept of C is -12
- $(x + 4)$ is a factor of $\mathrm{f}(x)$

find, in simplest form, $\mathrm{f}(x)$

(6)

$f(x) = \frac{6x^3}{3} + \frac{ax^2}{2} - 23x + c$

when $x = 0$ $y = -12$

$-12 = c$

$f(x) = 2x^3 + \frac{a}{2}x^2 - 23x - 12$

$(x+4)$ is a factor $\therefore$ $f(-4) = 0$

$2(-4)^3 + \frac{a}{2}(-4)^2 - 23(-4) - 12 = 0$

$-48 + 8a = 0$

$8a = 48$

$a = 6$

$f(x) = 2x^3 + 3x^2 - 23x - 12$

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 6 marks)

9. A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ°C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

(b) Use this information to evaluate the model.

(2)

when $t=0$ $\theta = 18$

$18 = A - B$ $[e^0 = 1]$

when $t=10$ $\theta = 44$

$44 = A - Be^{-0.07(10)}$

$A = 18 + B$ $A = 44 + Be^{-0.7}$

$18 + B = 44 + Be^{-0.7}$

$B - Be^{-0.7} = 26$

$B(1 - e^{-0.7}) = 26$

$B = \dfrac{26}{1 - e^{-0.7}}$

$= 51.6$

$18 = A - 51.6$

$A = 69.6$

$\theta = 69.6 - 51.6e^{-0.07t}$

b/ The Max temperature in the Model is 69.6°. The model is not accurate as 69.6° much less than 78° (not close)

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Question 9 continued

(Total for Question 9 is 6 marks)

10. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\cos 3A \equiv 4\cos^3 A - 3\cos A$$

(4)

(b) Hence solve, for $-90° \leqslant x \leqslant 180°$, the equation

$$1 - \cos 3x = \sin^2 x$$

(4)

a/ $\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= \cos A(\cos^2 A - \sin^2 A) - \sin A(2\sin A\cos A)$

$= \cos^3 A - \cos A \sin^2 A - 2\sin^2 A \cos A$

$= \cos^3 A - \cos A(1 - \cos^2 A) - 2\cos A(1 - \cos^2 A)$

$= \cos^3 A - \cos A + \cos^3 A - 2\cos A + 2\cos^3 A$

$= 4\cos^3 A - 3\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\sin 2A = 2\sin A\cos A$

$\sin^2 A = 1 - \cos^2 A$

b/ $1 - (4\cos^3 x - 3\cos x) = \sin^2 x$

$1 - 4\cos^3 x + 3\cos x = 1 - \cos^2 x$

$0 = 4\cos^3 x - \cos^2 x + 3\cos x$

$4\cos^3 x - \cos^2 x + 3\cos x = 0$

$\cos x(4\cos^2 x - \cos x + 3) = 0$

$\cos x(4\cos x + 3)(\cos x - 1) = 0$

$\cos x = 0$ $\quad$ $\cos x = -\frac{3}{4}$ $\quad$ $\cos x = 1$

$x = 90°, -90°$ $\quad$ $x = 138.6°, ~~221.4~~, ~~-138.6~~$ $\quad$ $x = 0°$

$x = -90°, 0°, 90°, 138.6°$

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 8 marks)

11.

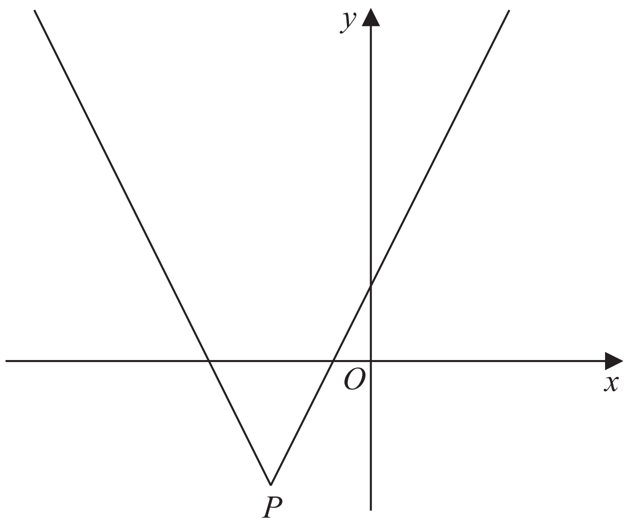

Figure 2

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point P, shown in Figure 2.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line l has equation $y = ax$, where a is a constant.

Given that l intersects $y = 2|x + 4| - 5$ at least once,

(c) find the range of possible values of a, writing your answer in set notation.

(3)

a/ $y = 2|x+4| - 5$

lowest point when $|x+4| = 0$

$x = -4$

when $x = -4$ $y = -5$

$(-4, -5)$

b/ $3x + 40 = 2|x+4| - 5$

$y = 2(x+4) - 5$ when $x \geqslant -4$ $y = -2(x+4) - 5$ when $x \leqslant -4$

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Question 11 continued

$3x + 40 = 2(x+4) - 5$

$3x + 40 = 2x + 8 - 5$

$3x + 40 = 2x + 3$

$x = -37$

$-37 < -4$ ∴ not a solution

$3x + 40 = -2(x+4) - 5$

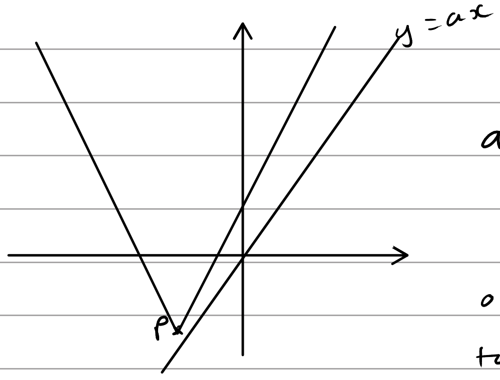
$3x + 40 = -2x - 8 - 5$

$3x + 40 = -2x - 13$

$5x = -53$

$x = \frac{-53}{5}$

c)



$a > 2$ (steeper gradient will meet $y = 2(x+4) - 5$)

or gradient must be low enough to meet P $(-4, -5)$

$(-4, -5)$ $(0, 0)$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5}{4}$

$a > 2$ or $a \leq \frac{5}{4}$

$\{a : a > 2\} \cup \{a : a \leq \frac{5}{4}\}$

Question 11 continued

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Question 11 continued

(Total for Question 11 is 7 marks)

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12.

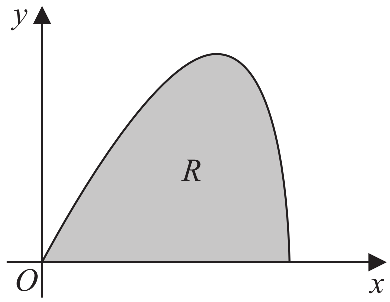

Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

(a) (i) Show that the area of R is given by $\displaystyle\int_0^{\frac{\pi}{2}} 60\sin t\cos^2 t\,\mathrm{d}t$

(3)

(ii) Hence show, by algebraic integration, that the area of R is exactly 20

(3)

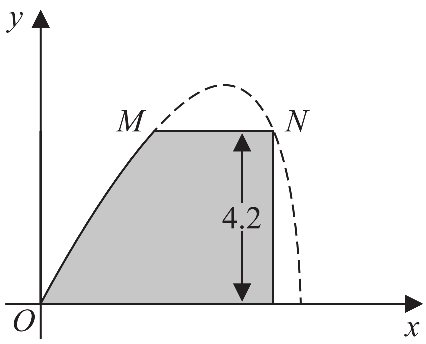

Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4.
Using the model and given that

- x and y are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width MN along the top of the dam

(b) calculate the width of the walkway.

(5)

a/ $\int y \, dx = \int y \frac{dx}{dt} \, dt$

crosses x when $y = 0$ $\quad 0 = 5 \sin 2t$

Question 12 continued

$0 = \sin 2t$

$2t = \sin^{-1}(0)$

$= 0, \pi$

$t = 0, \frac{\pi}{2}$

$\int_0^{\frac{\pi}{2}} y \frac{dx}{dt} \, dt$

$y = 5 \sin 2t$

$x = 6 \sin t$

$\frac{dx}{dt} = 6 \cos t$

$\int_0^{\frac{\pi}{2}} 5 \sin 2t \, (6 \cos t) \, dt$

$\int_0^{\frac{\pi}{2}} 30 \sin 2t \cos t \, dt$ $\quad \sin 2t = 2 \sin t \cos t$

$\int_0^{\frac{\pi}{2}} 30 \cos t \, (2 \sin t \cos t) \, dt$

$\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$

ii) $u = \cos t \quad \frac{du}{dt} = -\sin t$

$\int 60 \sin t \cos^2 t \frac{dt}{du} \, du$

$\int 60 \sin t \, u^2 \frac{1}{-\sin t} \, du$

$\int -60 u^2 \, du$

$\frac{-60 u^3}{3} = -20 u^3$

$= -20 \cos^3 t$

$\left[-20 \cos^3 t \right]_0^{\frac{\pi}{2}}$

$0 - -20 = \underline{20 \text{ unit}^2}$

Question 12 continued

b) when $y = 4.2$ $\quad 4.2 = 5\sin 2t$

$\frac{21}{25} = \sin 2t$

$2t = 0.997, 2.144$

$t = 0.499, 1.072$

when $t = 0.499$ $\quad x = 6\sin(0.499)$

$= 2.87$

when $t = 1.072$ $\quad x = 6\sin(1.072)$

$= 5.27$

$5.27 - 2.87 = \underline{\underline{2.4m}}$

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Question 12 continued

(Total for Question 12 is 11 marks)

13. The function g is defined by

$$g(x) = \frac{3\ln(x) - 7}{\ln(x) - 2} \qquad x > 0 \qquad x \neq k$$

where k is a constant.

(a) Deduce the value of k.

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of x in the domain of g.

(3)

(c) Find the range of values of a for which

$$g(a) > 0$$

(2)

a/ undefined when $\ln x - 2 = 0$

$\ln x = 2$

$x = e^2$

$k = e^2$

b/ $\dfrac{3\ln x - 7}{\ln x - 2}$ $\qquad u = 3\ln x - 7 \qquad v = \ln x - 2$

$\dfrac{du}{dx} = \dfrac{3}{x} \qquad \dfrac{dv}{dx} = \dfrac{1}{x}$

$$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$\frac{\frac{3}{x}(\ln x - 2) - \frac{1}{x}(3\ln x - 7)}{(\ln x - 2)^2}$$

$$\frac{\frac{3\ln x}{x} - \frac{6}{x} - \frac{3\ln x}{x} + \frac{7}{x}}{(\ln x - 2)^2}$$

Question 13 continued

$$\frac{\frac{1}{x}}{(\ln x - 2)^2}$$

$$\frac{1}{x(\ln x - 2)^2}$$

$x > 0$ x will be positive $(\ln x - 2)^2$ will always be positive

$$\therefore \frac{1}{x(\ln x - 2)^2} > 0$$

c/ $g(x) = \frac{3\ln x - 7}{\ln x - 2}$

$3\ln x - 7 > 0$

$\ln x > \frac{7}{3}$

$x > e^{\frac{7}{3}}$

$\ln x - 2 > 0$

$\ln x > 2$

$x > e^2$

Both positive when $x > e^{\frac{7}{3}}$

Both negative when $x < e^2$

also $x > 0$

$0 < x < e^2$ or $x > e^{\frac{7}{3}}$

Question 13 continued

Question 13 continued

(Total for Question 13 is 6 marks)

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Turn over

14. A circle C with radius r

- lies only in the 1st quadrant
- touches the x-axis and touches the y-axis

The line l has equation $2x + y = 12$

(a) Show that the x coordinates of the points of intersection of l with C satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that l is a tangent to C,

(b) find the two possible values of r, giving your answers as fully simplified surds.

(4)

a/ centre (r, r)

$$(x-r)^2 + (y-r)^2 = r^2$$

$$2x + y = 12$$
$$y = 12 - 2x$$

~~$(x-r)^2 + (12-2x-r)^2 = r^2$~~

$$(x-r)(x-r) + (y-r)(y-r) = r^2$$

$$x^2 - rx - rx + r^2 + y^2 - ry - ry + r^2 = r^2$$

$$x^2 - 2rx + y^2 - 2ry + 2r^2 = r^2$$

$$x^2 - 2rx + (12-2x)^2 - 2r(12-2x) + 2r^2 = r^2$$

$$x^2 - 2rx + (12-2x)(12-2x) - 24r + 4rx + 2r^2 = r^2$$

$$x^2 - 2rx + 144 - 24x - 24x + 4x^2 - 24r + 4rx + 2r^2 = r^2$$

$$5x^2 + 2rx - 48x + 144 - 24r + 2r^2 = r^2$$

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

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Question 14 continued

b) $b^2 - 4ac = 0$

$(2r-48)^2 - 4(5)(r^2-24r+144) = 0$

$(2r-48)(2r-48) - 20(r^2-24r+144) = 0$

$4r^2 - 96r - 96r + 2304 - 20r^2 + 480r - 2880 = 0$

$-16r^2 + 288r - 576 = 0$

$16r^2 - 288r + 576 = 0$

$r^2 - 18r + 36 = 0$

$r = \underline{9+3\sqrt{5}}$ or $r = \underline{9-3\sqrt{5}}$

P 6 6 7 8 6 A 0 4 3 5 2

Turn over ▶

Question 14 continued

Question 14 continued

(Total for Question 14 is 7 marks)

15. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

A geometric series has common ratio r and first term a.

Given $r \neq 1$ and $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

Given also that S_{10} is four times S_5

(b) find the exact value of r.

(4)

a/ $S_n = a + ar + ar^2 + ar^3 + \ldots + ar^{n-3} + ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + ar^4 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{10} = 4S_5$

$\frac{a(1-r^{10})}{1-r} = \frac{4a(1-r^5)}{1-r}$

$a(1-r^{10}) = 4a(1-r^5)$

$1-r^{10} = 4(1-r^5)$

$1-r^{10} = 4-4r^5$

$0 = r^{10} - 4r^5 + 3$

Question 15 continued

$0 = (r^5 - 3)(r^5 - 1)$

$r^5 = 3 \qquad r^5 = 1$

$r = \sqrt[5]{3} \qquad r = 1$

$\times$

P 6 6 7 8 6 A 0 4 7 5 2

Turn over ▶

Question 15 continued

Question 15 continued

(Total for Question 15 is 8 marks)

Positive integer

16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3

(4)

~~$(2n)^2 = 4n^2$~~ ~~$(2n+1)^2 = 4n^2 + 4n + 1$~~

$(3n)^2 = 9n^2$ $(3n+1)^2 = 9n^2 + 6n + 1$

$= 3(3n^2)$ $= 3(3n^2 + 2n) + 1$

$(3n+2)^2 = 9n^2 + 12n + 4$

$= 3(3n^2 + 4n + 1) + 1$

All natural numbers can be written as 3n, 3n+1 or 3n+2

In all cases the square is a multiple of 3 or 1 more than a multiple of 3.

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Question 16 continued

Question 16 continued

(Total for Question 16 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS