# Pearson Edexcel Level 3 GCE

| Paper reference | **9MA0/32** |

# Mathematics
**Advanced**
**PAPER 32: Mechanics**

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of $g$ is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

**1.** [*In this question, position vectors are given relative to a fixed origin.*]

At time $t$ seconds, where $t > 0$, a particle $P$ has velocity $\mathbf{v} \, \text{m s}^{-1}$ where

$$\mathbf{v} = 3t^2\mathbf{i} - 6t^{\frac{1}{2}}\mathbf{j}$$

(a) Find the speed of $P$ at time $t = 2$ seconds.

(2)

(b) Find an expression, in terms of $t$, $\mathbf{i}$ and $\mathbf{j}$, for the acceleration of $P$ at time $t$ seconds, where $t > 0$

(2)

At time $t = 4$ seconds, the position vector of $P$ is $(\mathbf{i} - 4\mathbf{j})\,\text{m}$.

(c) Find the position vector of $P$ at time $t = 1$ second.

(4)

a/    when $t = 2$    $v = 3(2)^2 \, \mathbf{i} - 6(2)^{\frac{1}{2}} \, \mathbf{j}$

$= 12\mathbf{i} - 6\sqrt{2} \, \mathbf{j}$

$speed = \sqrt{12^2 + (6\sqrt{2})^2}$

$= 6\sqrt{6} \quad \text{ms}^{-1}$

b/    $a = \dfrac{dv}{dt}$

$= 6t \, \mathbf{i} - 3t^{-\frac{1}{2}} \, \mathbf{j} \qquad \text{ms}^{-2}$

$s = \int v \, dt$

$= \int 3t^2 \, \mathbf{i} - 6t^{\frac{1}{2}} \, \mathbf{j}$

$= t^3 \mathbf{i} - 4t^{\frac{3}{2}} \, \mathbf{j} + c \qquad \text{when } t = 4$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad s = \mathbf{i} - 4\mathbf{j}$

$\mathbf{i} - 4\mathbf{j} = 64\mathbf{i} - 32\mathbf{j} + c$

$c = -63\mathbf{i} + 28\mathbf{j}$

$s = t^3 \mathbf{i} - 4t^{\frac{3}{2}} \mathbf{j} - 63\mathbf{i} + 28\mathbf{j}$

$$s = \left(t^3 - 63\right)i + \left(28 - 4t^{3/2}\right)j$$

when $t = 1$ $\qquad s = -\underline{62i + 24j}$

**2.**
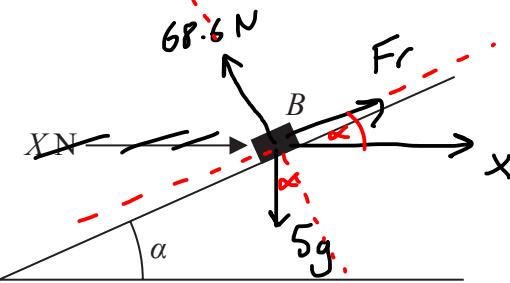


**Figure 1**

A rough plane is inclined to the horizontal at an angle $\alpha$, where $\tan\alpha = \dfrac{3}{4}$

$\cos\alpha = 4/5$

$\sin\alpha = 3/5$

A small block $B$ of mass $5\,\text{kg}$ is held in equilibrium on the plane by a horizontal force of magnitude $X$ newtons, as shown in Figure 1.
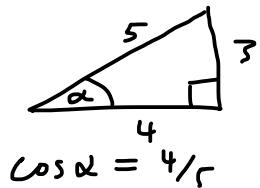
The force acts in a vertical plane which contains a line of greatest slope of the inclined plane.

The block $B$ is modelled as a particle.

The magnitude of the normal reaction of the plane on $B$ is $68.6\,\text{N}$.

Using the model,

(a) (i) find the magnitude of the frictional force acting on $B$,

**(3)**

  (ii) state the direction of the frictional force acting on $B$.

**(1)**

The horizontal force of magnitude $X$ newtons is now removed and $B$ moves down the plane.

Given that the coefficient of friction between $B$ and the plane is 0.5

(b) find the acceleration of $B$ down the plane.

**(6)**

a/ Resolving perp. to plane

$$68.6 = 5g\cos\alpha + X\sin\alpha$$

$$68.6 = 5g\,(4/5) + (3/5)\,X$$

$$68.6 - 4g = 3/5\,X$$

$$X = 49\,N$$

Resolving parallel to the plane

$$F_r + 49 \cos \alpha = 5g \sin \alpha$$

$$F_r + 49 \left(\frac{4}{5}\right) = 5g \left(\frac{3}{5}\right)$$

$$F_r = -9.8 \, N$$

$$\therefore \quad F_r = 9.8 \, N \text{ acting } \underline{down} \text{ the plane}$$

**Question 2 continued**

(Total for Question 2 is 10 marks)

**3.** [*In this question,* **i** *and* **j** *are horizontal unit vectors.*]

A particle $P$ of mass $4\,\text{kg}$ is at rest at the point $A$ on a smooth horizontal plane.

At time $t = 0$, two forces, $\mathbf{F}_1 = (4\mathbf{i} - \mathbf{j})\,\text{N}$ and $\mathbf{F}_2 = (\lambda\mathbf{i} + \mu\mathbf{j})\,\text{N}$, where $\lambda$ and $\mu$ are constants, are applied to $P$

Given that $P$ moves in the direction of the vector $(3\mathbf{i} + \mathbf{j})$

(a) show that

$$\lambda - 3\mu + 7 = 0$$

**(4)**

At time $t = 4$ seconds, $P$ passes through the point $B$.

Given that $\lambda = 2$

(b) find the length of $AB$.

**(5)**

a/ $\quad F_1 + F_2 = x \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} \lambda \\ \mu \end{pmatrix} = x \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$4 + \lambda = 3x$

$-1 + \mu = x \quad (\times 3)$

$-3 + 3\mu = 3x$

$4 + \lambda = -3 + 3\mu$

$\lambda - 3\mu + 7 = 0$

b/ $\quad \lambda = 2 \qquad 2 - 3\mu + 7 = 0$

$9 = 3\mu$

$\mu = 3$

$\therefore \quad F_1 + F_2 = \begin{pmatrix} 4 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$

$F = ma$

$$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = 4a$$

$$a = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$$

$s =$
$u = 0$
$v$
$a = \begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$
$t = 4$

$$s = ut + \frac{1}{2}at^2$$

$$= 0(4) + \frac{1}{2}\begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}(4)^2$$

$$= 8\begin{pmatrix} 1.5 \\ 0.5 \end{pmatrix}$$

$$= \begin{pmatrix} 12 \\ 4 \end{pmatrix}$$

$$distance = \sqrt{12^2 + 4^2}$$

$$= 4\sqrt{10} \ m$$

P 7 2 1 3 1 A 0 1 0 2 0

**(Total for Question 3 is 9 marks)**

**4.**
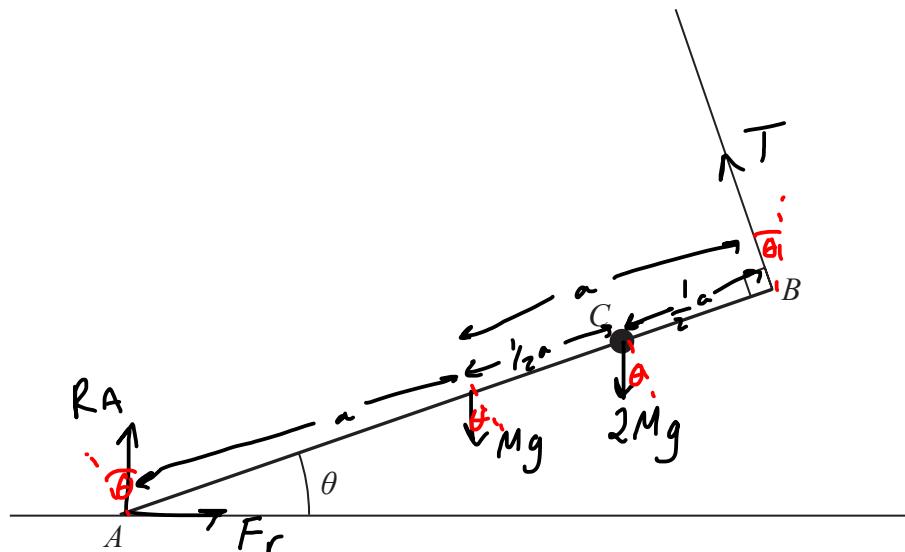


**Figure 2**

A uniform rod $AB$ has mass $M$ and length $2a$

A particle of mass $2M$ is attached to the rod at the point $C$, where $AC = 1.5a$

The rod rests with its end $A$ on rough horizontal ground.

The rod is held in equilibrium at an angle $\theta$ to the ground by a light string that is attached to the end $B$ of the rod.

The string is perpendicular to the rod, as shown in Figure 2.

(a) Explain why the frictional force acting on the rod at $A$ acts horizontally to the right on the diagram.

**(1)**

The tension in the string is $T$

(b) Show that $T = 2Mg\cos\theta$

**(3)**

Given that $\cos\theta = \dfrac{3}{5}$   $\sin\theta = \dfrac{4}{5}$

(c) show that the magnitude of the vertical force exerted by the ground on the rod at $A$ is $\dfrac{57Mg}{25}$

**(3)**

The coefficient of friction between the rod and the ground is $\mu$

Given that the rod is in limiting equilibrium,

(d) show that $\mu = \dfrac{8}{19}$

**(4)**

a) It opposes the motion, the only other horizontal force acts to the left.

b) Taking moments about A:

$$a \, Mg \cos\theta + \frac{3}{2}a \cdot 2Mg \cos\theta = 2a \, T$$

$$Mg \cos\theta + 3Mg \cos\theta = 2T$$

$$4Mg \cos\theta = 2T$$

$$2Mg \cos\theta = T$$

c) $\cos\theta = \frac{3}{5}$ ∴ $T = \frac{6}{5}Mg$

Forces up = Forces down

$$T \cos\theta + R_A = 3Mg$$

$$\frac{6}{5}Mg\left(\frac{3}{5}\right) + R_A = 3Mg$$

$$\frac{18}{25}Mg + R_A = 3Mg$$

$$R_A = \frac{57}{25}Mg$$

d) Fr takes max value $= \mu R_A$

Forces left = Forces right

$$T \sin\theta = \mu R_A$$

$$\frac{6}{5}Mg\left(\frac{4}{5}\right) = \mu \frac{57}{25}Mg$$

$$\frac{24}{25} = \frac{57}{25}\mu$$

$$\mu = \frac{24}{57} = \frac{8}{19}$$

**Question 4 continued**

**(Total for Question 4 is 11 marks)**
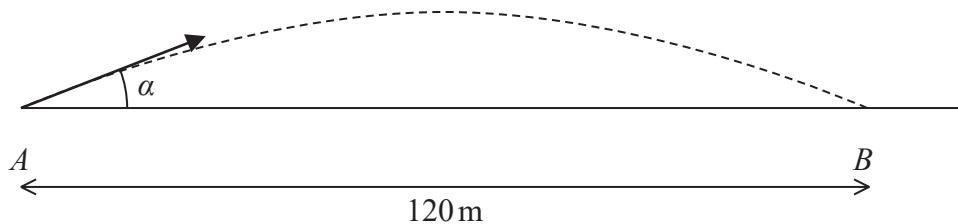
**Figure 3**

A golf ball is at rest at the point $A$ on horizontal ground.

The ball is hit and initially moves at an angle $\alpha$ to the ground.

The ball first hits the ground at the point $B$, where $AB = 120\,\text{m}$, as shown in Figure 3.

The motion of the ball is modelled as that of a particle, moving freely under gravity, whose initial speed is $U\,\text{ms}^{-1}$

Using this model,

(a) show that $U^2 \sin\alpha\cos\alpha = 588$

**(6)**

The ball reaches a maximum height of $10\,\text{m}$ above the ground.

(b) Show that $U^2 = 1960$

**(4)**

In a refinement to the model, the effect of air resistance is included.

The motion of the ball, from $A$ to $B$, is now modelled as that of a particle whose initial speed is $V\,\text{ms}^{-1}$

This refined model is used to calculate a value for $V$

(c) State which is greater, $U$ or $V$, giving a reason for your answer.

**(1)**

(d) State one further refinement to the model that would make the model more realistic.

**(1)**

a/ Horizontally

$s = 120$
$u = U\cos\alpha$
$v = U\cos\alpha$
$a = 0$
$t = T$

Vertically

$s = 0$
$u = U\sin\alpha$
$v =$
$a = -9.8$
$t = T$

$s = ut + \frac{1}{2}at^2$
$120 = (U\cos\alpha)T$

$0 = (U\sin\alpha)T + \frac{1}{2}(-9.8)T^2$

$$T = \frac{120}{u \cos \alpha}$$

$$0 = u \sin \alpha \cdot \frac{120}{u \cos \alpha} + \frac{1}{2}(-9.8)\left(\frac{120}{u \cos \alpha}\right)^2$$

$$0 = \frac{120 u \sin \alpha}{u \cos \alpha} - 4.9 \left(\frac{14400}{u^2 \cos^2 \alpha}\right)$$

$$0 = 120 \sin \alpha - 4.9 \left(\frac{14400}{u^2 \cos \alpha}\right)$$

$$0 = 120 u^2 \cos \alpha \sin \alpha - 70560$$

$$0 = u^2 \cos \alpha \sin \alpha - 588$$

$$\underline{u^2 \cos \alpha \sin \alpha = 588}$$

b/  Max height = 10      Vertically

when at Max  v = 0

$S = 10$
$u = u \sin \alpha$
$v = 0$
$a = -9.8$
$t = T.$

$$v^2 = u^2 + 2as$$

$$0 = (u \sin \alpha)^2 + 2(-9.8)(10)$$

$$0 = u^2 \sin^2 \alpha - 196$$

$$u^2 \sin^2 \alpha = 196 \qquad u^2 \cos \alpha \sin \alpha = 588$$

$$\frac{u^2 \sin^2 \alpha}{u^2 \cos \alpha \sin \alpha} = \frac{196}{588}$$

$$\tan \alpha = \frac{1}{3}$$
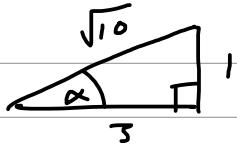


$$\cos \alpha = \frac{3}{\sqrt{10}} \qquad \sin \alpha = \frac{1}{\sqrt{10}}$$

$$u^2 \cos \alpha \sin \alpha = 588$$

$$u^2 \left(\frac{3}{\sqrt{10}}\right)\left(\frac{1}{\sqrt{10}}\right) = 588$$

$$\frac{3}{10} u^2 = 588$$

$$u^2 = 1960$$

c/ ✓ because of the air resistance

d/ The ball could not be modelled as a particle. (size/shape/dimensions of the ball included)

**Question 5 continued**

**(Total for Question 5 is 12 marks)**

**TOTAL FOR MECHANICS IS 50 MARKS**

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