

Paper 1: Core Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1	$\frac{1}{(r+1)(r+3)} \equiv \frac{A}{(r+1)} + \frac{B}{(r+3)} \Rightarrow A = \dots, B = \dots$	M1	3.1a
	$\sum_{r=1}^n \frac{1}{(r+1)(r+3)} =$ $\frac{1}{2 \times 2} - \frac{1}{2 \times 4} + \frac{1}{2 \times 3} - \frac{1}{2 \times 5} + \dots + \frac{1}{2n} - \frac{1}{2(n+2)} + \frac{1}{2(n+1)} - \frac{1}{2(n+3)}$	M1	2.1
	$= \frac{1}{4} + \frac{1}{6} - \frac{1}{2(n+2)} - \frac{1}{2(n+3)}$	A1	2.2a
	$= \frac{5(n+2)(n+3) - 6(n+3) - 6(n+2)}{12(n+2)(n+3)}$	M1	1.1b
	$= \frac{n(5n+13)}{12(n+2)(n+3)}$	A1	1.1b
	(5)		
	Alternative by induction: $n=1 \Rightarrow \frac{1}{8} = \frac{a+b}{12 \times 3 \times 4}, n=2 \Rightarrow \frac{1}{8} + \frac{1}{15} = \frac{2(2a+b)}{12 \times 4 \times 5}$ $a+b=18, 2a+b=23 \Rightarrow a = \dots, b = \dots$	M1	3.1a
	Assume true for $n = k$ so $\sum_{r=1}^k \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)}$		
	$\sum_{r=1}^{k+1} \frac{1}{(r+1)(r+3)} = \frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)}$	M1	2.1
	$\frac{k(5k+13)}{12(k+2)(k+3)} + \frac{1}{(k+2)(k+4)} = \frac{k(5k+13)(k+4) + 12(k+3)}{12(k+2)(k+3)(k+4)}$	A1	2.2a
	$= \frac{5k^3 + 33k^2 + 52k + 12k + 36}{12(k+2)(k+3)(k+4)} = \frac{(k+1)(k+2)(5k+18)}{12(k+2)(k+3)(k+4)}$	M1	1.1b
	$= \frac{(k+1)(5(k+1)+13)}{12(\underline{k+1}+2)(\underline{k+1}+3)}$ So true for $n = k + 1$	A1	1.1b
	So $\sum_{r=1}^n \frac{1}{(r+1)(r+3)} = \frac{n(5n+13)}{12(n+2)(n+3)}$		
(5)			
(5 marks)			

Question 1 notes:**Main Scheme**

M1: Valid attempt at partial fractions

M1: Starts the process of differences to identify the relevant fractions at the start and end

A1: Correct fractions that do not cancel

M1: Attempt common denominator

A1: Correct answer

Alternative by Induction:

M1: Uses $n = 1$ and $n = 2$ to identify values for a and b

M1: Starts the induction process by adding the $(k + 1)^{\text{th}}$ term to the sum of k terms

A1: Correct single fraction

M1: Attempt to factorise the numerator

A1: Correct answer and conclusion

Question	Scheme	Marks	AOs
2	When $n = 1$, $2^{3n+1} + 3(5^{2n+1}) = 16 + 375 = 391$ $391 = 17 \times 23$ so the statement is true for $n = 1$	B1	2.2a
	Assume true for $n = k$ so $2^{3k+1} + 3(5^{2k+1})$ is divisible by 17	M1	2.4
	$f(k+1) - f(k) = 2^{3k+4} + 3(5^{2k+3}) - 2^{3k+1} - 3(5^{2k+1})$	M1	2.1
	$= 7 \times 2^{3k+1} + 7 \times 3(5^{2k+1}) + 17 \times 3(5^{2k+1})$		
	$= 7f(k) + 17 \times 3(5^{2k+1})$	A1	1.1b
	$f(k+1) = 8f(k) + 17 \times 3(5^{2k+1})$	A1	1.1b
	If the statement is true for $n = k$ then it has been shown true for $n = k + 1$ and as it is true for $n = 1$, the statement is true for all positive integers n	A1	2.4
		(6)	
(6 marks)			
Notes:			
B1: Shows the statement is true for $n = 1$			
M1: Assumes the statement is true for $n = k$			
M1: Attempts $f(k+1) - f(k)$			
A1: Correct expression in terms of $f(k)$			
A1: Correct expression in terms of $f(k)$			
A1: Obtains a correct expression for $f(k + 1)$			
A1: Correct complete conclusion			

Question	Scheme	Marks	AOs
3	$z = 3 - 2i$ is also a root	B1	1.2
	$(z - (3 + 2i))(z - (3 - 2i)) = \dots$ or Sum of roots = 6, Product of roots = 13 $\Rightarrow \dots$	M1	3.1a
	$= z^2 - 6z + 13$	A1	1.1b
	$(z^4 + az^3 + 6z^2 + bz + 65) = (z^2 - 6z + 13)(z^2 + cz + 5) \Rightarrow c = \dots$	M1	3.1a
	$z^2 + 2z + 5 = 0$	A1	1.1b
	$z^2 + 2z + 5 = 0 \Rightarrow z = \dots$	M1	1.1a
	$z = -1 \pm 2i$	A1	1.1b
		B1 $3 \pm 2i$ Plotted correctly	1.1b
	B1ft $-1 \pm 2i$ Plotted correctly	1.1b	
(9 marks)			
Notes:			
B1: Identifies the complex conjugate as another root M1: Uses the conjugate pair and a correct method to find a quadratic factor A1: Correct quadratic M1: Uses the given quartic and their quadratic to identify the value of c A1: Correct 3TQ M1: Solves their second quadratic A1: Correct second conjugate pair B1: First conjugate pair plotted correctly and labelled B1ft: Second conjugate pair plotted correctly and labelled (Follow through their second conjugate pair)			

Question	Scheme	Marks	AOs
4	$4 + \cos 2\theta = \frac{9}{2} \Rightarrow \theta = \dots$	M1	3.1a
	$\theta = \frac{\pi}{6}$	A1	1.1b
	$\frac{1}{2} \int (4 + \cos 2\theta)^2 d\theta = \frac{1}{2} \int (16 + 8 \cos 2\theta + \cos^2 2\theta) d\theta$	M1	3.1a
	$\cos^2 2\theta = \frac{1}{2} + \frac{1}{2} \cos 4\theta \Rightarrow A = \frac{1}{2} \int \left(16 + 8 \cos 2\theta + \frac{1}{2} + \frac{1}{2} \cos 4\theta \right) d\theta$	M1	3.1a
	$= \frac{1}{2} \left[16\theta + 4 \sin 2\theta + \frac{\sin 4\theta}{8} + \frac{\theta}{2} \right]$	A1	1.1b
	Using limits 0 and their $\frac{\pi}{6}$: $\frac{1}{2} \left[\frac{33\pi}{12} + 2\sqrt{3} + \frac{\sqrt{3}}{16} - (0) \right]$	M1	1.1b
	Area of triangle = $\frac{1}{2} (r \cos \theta)(r \sin \theta) = \frac{1}{2} \times \frac{81}{4} \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$	M1	3.1a
	Area of R = $\frac{33\pi}{24} + \frac{33\sqrt{3}}{32} - \frac{81\sqrt{3}}{32}$	M1	1.1b
	$= \frac{11}{8} \pi - \frac{3\sqrt{3}}{2} \left(p = \frac{11}{8}, q = -\frac{3}{2} \right)$	A1	1.1b

(9 marks)

Notes:

- M1:** Realises the angle for A is required and attempts to find it
- A1:** Correct angle
- M1:** Uses a correct area formula and squares r to achieve a 3TQ integrand in $\cos 2\theta$
- M1:** Use of the correct double angle identity on the integrand to achieve a suitable form for integration
- A1:** Correct integration
- M1:** Correct use of limits
- M1:** Identifies the need to subtract the area of a triangle and so finds the area of the triangle
- M1:** Complete method for the area of R
- A1:** Correct final answer

Question	Scheme	Marks	AOs
5(a)	Pond contains $1000 + 5t$ litres after t days	M1	3.3
	If x is the amount of pollutant in the pond after t days		
	Rate of pollutant out = $20 \times \frac{x}{1000+5t}$ g per day	M1	3.3
	Rate of pollutant in = 25×2 g = 50g per day	B1	2.2a
	$\frac{dx}{dt} = 50 - \frac{4x}{200+t}$ *	A1*	1.1b
	(4)		
(b)	$I = e^{\int \frac{4}{200+t} dt} = (200+t)^4 \Rightarrow x(200+t)^4 = \int 50(200+t)^4 dt$	M1	3.1b
	$x(200+t)^4 = 10(200+t)^5 + c$	A1	1.1b
	$x = 0, t = 0 \Rightarrow c = -3.2 \times 10^{12}$	M1	3.4
	$t = 8 \Rightarrow x = 10(200+8) - \frac{3.2 \times 10^{12}}{(200+8)^4}$	M1	1.1b
	$= 370\text{g}$	A1	2.2b
	(5)		
(c)	e.g. <ul style="list-style-type: none"> The model should take into account the fact that the pollutant does not dissolve throughout the pond upon entry The rate of leaking could be made to vary with the volume of water in the pond 	B1	3.5c
		(1)	
(10 marks)			
Notes:			
(a)	M1: Forms an expression of the form $1000 + kt$ for the volume of water in the pond at time t M1: Expresses the amount of pollutant out in terms of x and t B1: Correct interpretation for pollutant entering the pond A1*: Puts all the components together to form the correct differential equation		
(b)	M1: Uses the model to find the integrating factor and attempts solution of their differential equation A1: Correct solution M1: Interprets the initial conditions to find the constant of integration M1: Uses their solution to the problem to find the amount of pollutant after 8 days A1: Correct number of grams		
(c)	B1: Suggests a suitable refinement to the model		

Question	Scheme	Marks	AOs
6(a)	$f(x) = \frac{x+2}{x^2+9} = \frac{x}{x^2+9} + \frac{2}{x^2+9}$	B1	3.1a
	$\int \frac{x}{x^2+9} dx = k \ln(x^2+9) (+c)$	M1	1.1b
	$\int \frac{2}{x^2+9} dx = k \arctan\left(\frac{x}{3}\right) (+c)$	M1	1.1b
	$\int \frac{x+2}{x^2+9} dx = \frac{1}{2} \ln(x^2+9) + \frac{2}{3} \arctan\left(\frac{x}{3}\right) + c$	A1	1.1b
		(4)	
(b)	$\int_0^3 f(x) dx = \left[\frac{1}{2} \ln(x^2+9) + \frac{2}{3} \arctan\left(\frac{x}{3}\right) \right]_0^3$ $= \frac{1}{2} \ln 18 + \frac{2}{3} \arctan\left(\frac{3}{3}\right) - \left(\frac{1}{2} \ln 9 + \frac{2}{3} \arctan(0) \right)$ $= \frac{1}{2} \ln \frac{18}{9} + \frac{2}{3} \arctan\left(\frac{3}{3}\right)$	M1	1.1b
	Mean value = $\frac{1}{3-0} \left(\frac{1}{2} \ln 2 + \frac{\pi}{6} \right)$	M1	2.1
	$\frac{1}{6} \ln 2 + \frac{1}{18} \pi^*$	A1*	2.2a
		(3)	
(c)	$\frac{1}{6} \ln 2 + \frac{1}{18} \pi + \ln k$	M1	2.2a
	$\frac{1}{6} \ln 2k^6 + \frac{1}{18} \pi$	A1	1.1b
		(2)	
(9 marks)			
Notes:			
(a)			
B1: Splits the fraction into two correct separate expressions			
M1: Recognises the required form for the first integration			
M1: Recognises the required form for the second integration			
A1: Both expressions integrated correctly and added together with constant of integration included			
(b)			
M1: Uses limits correctly and combines logarithmic terms			
M1: Correctly applies the method for the mean value for their integration			
A1*: Correct work leading to the given answer			
(c)			
M1: Realises that the effect of the transformation is to increase the mean value by $\ln k$			
A1: Combines \ln 's correctly to obtain the correct expression			

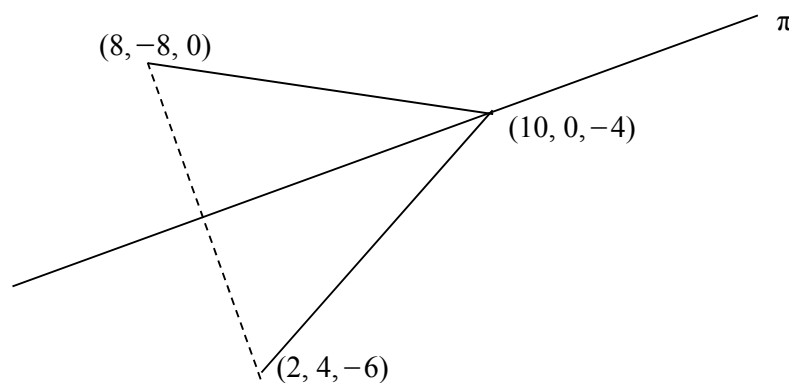
Question	Scheme	Marks	AOs
7(a)	$x = \cos \theta + \sin \theta \cos \theta = -y \cos \theta$	M1	2.1
	$\sin \theta = -y - 1$	M1	2.1
	$\left(\frac{x}{-y}\right)^2 = 1 - (-y - 1)^2$	M1	2.1
	$x^2 = -(y^4 + 2y^3)^*$	A1*	1.1b
		(4)	
(b)	$V = \pi \int x^2 dy = \pi \int -(y^4 + 2y^3) dy$	M1	3.4
	$= \pi \left[-\left(\frac{y^5}{5} + \frac{y^4}{2}\right) \right]$	A1	1.1b
	$= -\pi \left[\left(\frac{(0)^5}{5} + \frac{(0)^4}{2}\right) - \left(\frac{(-2)^5}{5} + \frac{(-2)^4}{2}\right) \right]$	M1	3.4
	$= 1.6\pi \text{ cm}^3 \text{ or awrt } 5.03 \text{ cm}^3$	A1	1.1b
		(4)	
(8 marks)			
Notes:			
(a)			
M1: Obtains x in terms of y and $\cos \theta$			
M1: Obtains an equation connecting y and $\sin \theta$			
M1: Uses Pythagoras to obtain an equation in x and y only			
A1*: Obtains printed answer			
(b)			
M1: Uses the correct volume of revolution formula with the given expression			
A1: Correct integration			
M1: Correct use of correct limits			
A1: Correct volume			

Question	Scheme	Marks	AOs
8	$2 + 4\lambda - 2(4 - 2\lambda) - 6 + \lambda = 6 \Rightarrow \lambda = \dots$	M1	1.1b
	$\lambda = 2 \Rightarrow$ Required point is $(2 + 2(4), 4 + 2(-2), -6 + 2(1))$ $(10, 0, -4)$	A1	1.1b
	$2 + t - 2(4 - 2t) - 6 + t = 6 \Rightarrow t = \dots$	M1	3.1a
	$t = 3$ so reflection of $(2, 4, -6)$ is $(2 + 6(1), 4 + 6(-2), -6 + 6(1))$ $(8, -8, 0)$	M1	3.1a
		A1	1.1b
	$\begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} - \begin{pmatrix} 8 \\ -8 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ -4 \end{pmatrix}$	M1	3.1a
	$\mathbf{r} = \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} + k \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$ or equivalent e.g. $\left(\mathbf{r} - \begin{pmatrix} 10 \\ 0 \\ -4 \end{pmatrix} \right) \times \begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \mathbf{0}$	A1	2.5
	(7)		

(7 marks)

Notes:

- M1:** Substitutes the parametric equation of the line into the equation of the plane and solves for λ
- A1:** Obtains the correct coordinates of the intersection of the line and the plane
- M1:** Substitutes the parametric form of the line perpendicular to the plane passing through $(2, 4, -6)$ into the equation of the plane to find t
- M1:** Find the reflection of $(2, 4, -6)$ in the plane
- A1:** Correct coordinates
- M1:** Determines the direction of l by subtracting the appropriate vectors
- A1:** Correct vector equation using the correct notation



Question	Scheme	Marks	AOs
9(a)(i)	Weight = mass \times g $\Rightarrow m = \frac{30000}{g} = 3000$ But mass is in thousands of kg, so $m = 3$	M1	3.3
(ii)	$\frac{dx}{dt} = 40 \cos t + 20 \sin t, \frac{d^2x}{dt^2} = -40 \sin t + 20 \cos t$	M1	1.1b
	$3(-40 \sin t + 20 \cos t) + 4(40 \cos t + 20 \sin t) + 40 \sin t - 20 \cos t = \dots$	M1	1.1b
	$= 200 \cos t$ so PI is $x = 40 \sin t - 20 \cos t$	A1*	2.1
	or		
	Let $x = a \cos t + b \sin t$ $\frac{dx}{dt} = -a \sin t + b \cos t, \frac{d^2x}{dt^2} = -a \cos t - b \sin t$	M1	1.1b
	$4b - 2a = 200, -2b - 4a = 0 \Rightarrow a = \dots, b = \dots$	M1	2.1
	$x = 40 \sin t - 20 \cos t$	A1*	1.1b
(iii)	$3\lambda^2 + 4\lambda + 1 = 0 \Rightarrow \lambda = -1, -\frac{1}{3}$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t}$	A1	1.1b
	$x = PI + CF$	M1	1.1b
	$x = Ae^{-t} + Be^{-\frac{1}{3}t} + 40 \sin t - 20 \cos t$	A1	1.1b
	(8)		
(b)	$t = 0, x = 0 \Rightarrow A + B = 20$	M1	3.4
	$x = 0, \frac{dx}{dt} = -Ae^{-t} - \frac{1}{3}Be^{-\frac{1}{3}t} + 40 \cos t + 20 \sin t = 0$ $\Rightarrow A + \frac{1}{3}B = 40$	M1	3.4
	$x = 50e^{-t} - 30e^{-\frac{1}{3}t} + 40 \sin t - 20 \cos t$	A1	1.1b
	$t = 9 \Rightarrow x = 33\text{m}$	A1	3.4
	(4)		
(12 marks)			

Question 9 notes:
<p>(a)(i) M1: Correct explanation that in the model, $m = 3$</p>
<p>(ii) M1: Differentiates the given PI twice M1: Substitutes into the given differential equation A1*: Reaches $200\cos t$ and makes a conclusion or M1: Uses the correct form for the PI and differentiates twice M1: Substitutes into the given differential equation and attempts to solve A1*: Correct PI</p>
<p>(iii) M1: Uses the model to form and solve the auxiliary equation A1: Correct complementary function M1: Uses the correct notation for the general solution by combining PI and CF A1: Correct General Solution for the model</p>
<p>(b) M1: Uses the initial conditions of the model, $t = 0$ at $x = 0$, to form an equation in A and B M1: Uses $\frac{dx}{dt} = 0$ at $x = 0$ in the model to form an equation in A and B A1: Correct PS A1: Obtains 33m using the assumptions made in the model</p>

