



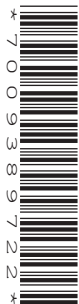
Oxford Cambridge and RSA

# AS Level Mathematics A

H230/02 Pure Mathematics and Mechanics

**Wednesday 23 May 2018 – Morning**

**Time allowed: 1 hour 30 minutes**



**You must have:**

- Printed Answer Booklet

**You may use:**

- a scientific or graphical calculator

## INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $g \text{ m s}^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

## INFORMATION

- The total number of marks for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

**Formulae**  
**AS Level Mathematics A (H230)**

**Binomial series**

$$(a + b)^n = a^n + {}^n C_1 a^{n-1} b + {}^n C_2 a^{n-2} b^2 + \dots + {}^n C_r a^{n-r} b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^n C_r = {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Standard deviation**

$$\sqrt{\frac{\sum(x-\bar{x})^2}{n}} = \sqrt{\frac{\sum x^2}{n} - \bar{x}^2} \quad \text{or} \quad \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}} = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$$

**The binomial distribution**

If  $X \sim B(n, p)$  then  $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ , Mean of  $X$  is  $np$ , Variance of  $X$  is  $np(1-p)$

**Kinematics**

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

## Section A: Pure Mathematics

Answer **all** the questions

- 1 In triangle  $ABC$ ,  $AB = 20$  cm and angle  $B = 45^\circ$ .
- (i) Given that  $AC = 16$  cm, find the two possible values for angle  $C$ , correct to 1 decimal place. [4]
- (ii) Given instead that the area of the triangle is  $75\sqrt{2}$  cm<sup>2</sup>, find  $BC$ . [2]
- 2 (i) The curve  $y = \frac{2}{3+x}$  is translated by four units in the positive  $x$ -direction. State the equation of the curve after it has been translated. [2]
- (ii) Describe fully the single transformation that transforms the curve  $y = \frac{2}{3+x}$  to  $y = \frac{5}{3+x}$ . [2]

- 3 In each of the following cases choose one of the statements

$$P \Rightarrow Q \quad P \Leftarrow Q \quad P \Leftrightarrow Q$$

to describe the relationship between  $P$  and  $Q$ .

- (i)  $P: y = 3x^5 - 4x^2 + 12x$   
 $Q: \frac{dy}{dx} = 15x^4 - 8x + 12$  [1]
- (ii)  $P: x^5 - 32 = 0$  where  $x$  is real  
 $Q: x = 2$  [1]
- (iii)  $P: \ln y < 0$   
 $Q: y < 1$  [1]
- 4 (i) Express  $4x^2 - 12x + 11$  in the form  $a(x+b)^2 + c$ . [3]
- (ii) State the number of real roots of the equation  $4x^2 - 12x + 11 = 0$ . [1]
- (iii) Explain fully how the value of  $r$  is related to the number of real roots of the equation  $p(x+q)^2 + r = 0$  where  $p, q$  and  $r$  are real constants and  $p > 0$ . [2]

- 5 In this question you must show detailed reasoning.

The line  $x + 5y = k$  is a tangent to the curve  $x^2 - 4y = 10$ . Find the value of the constant  $k$ . [5]

- 6 A pan of water is heated until it reaches  $100^{\circ}\text{C}$ . Once the water reaches  $100^{\circ}\text{C}$ , the heat is switched off and the temperature  $T^{\circ}\text{C}$  of the water decreases. The temperature of the water is modelled by the equation

$$T = 25 + ae^{-kt},$$

where  $t$  denotes the time, in minutes, after the heat is switched off and  $a$  and  $k$  are positive constants.

- (i) Write down the value of  $a$ . [1]  
 (ii) Explain what the value of 25 represents in the equation  $T = 25 + ae^{-kt}$ . [1]

When the heat is switched off, the initial rate of decrease of the temperature of the water is  $15^{\circ}\text{C}$  per minute.

- (iii) Calculate the value of  $k$ . [3]  
 (iv) Find the time taken for the temperature of the water to drop from  $100^{\circ}\text{C}$  to  $45^{\circ}\text{C}$ . [3]  
 (v) A second pan of water is heated, but the heat is turned off when the water is at a temperature of less than  $100^{\circ}\text{C}$ . Suggest how the equation for the temperature as the water cools would be modified by this. [1]

- 7 (i) Show that the equation

$$2 \sin x \tan x = \cos x + 5$$

can be expressed in the form

$$3 \cos^2 x + 5 \cos x - 2 = 0. \quad [3]$$

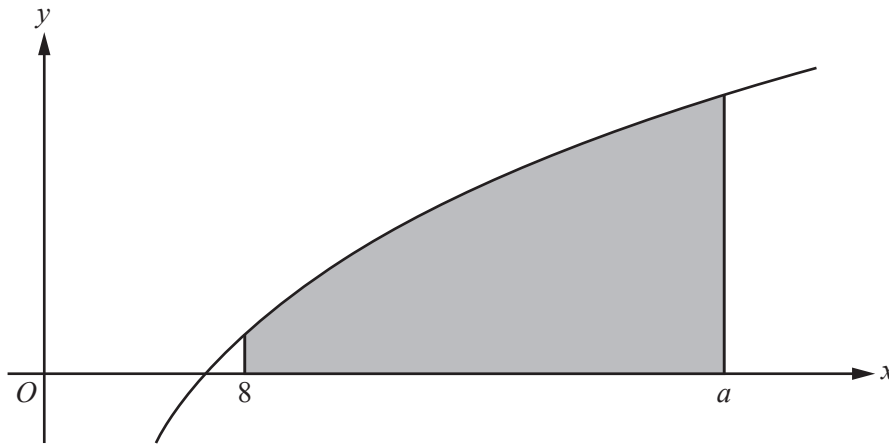
- (ii) Hence solve the equation

$$2 \sin 2\theta \tan 2\theta = \cos 2\theta + 5,$$

giving all values of  $\theta$  between  $0^{\circ}$  and  $180^{\circ}$ , correct to 1 decimal place. [5]

**8 In this question you must show detailed reasoning.**

The diagram shows part of the graph of  $y = 2x^{\frac{1}{3}} - \frac{7}{x^{\frac{1}{3}}}$ . The shaded region is enclosed by the curve, the  $x$ -axis and the lines  $x = 8$  and  $x = a$ , where  $a > 8$ .



Given that the area of the shaded region is 45 square units, find the value of  $a$ .

[9]

**Section B: Mechanics**  
Answer **all** the questions

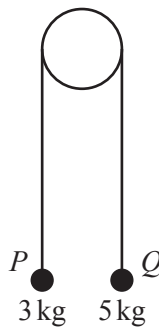
- 9 In this question the horizontal unit vectors  $\mathbf{i}$  and  $\mathbf{j}$  are in the directions east and north respectively.

A model ship of mass 2 kg is moving so that its acceleration vector  $\mathbf{a} \text{ ms}^{-2}$  at time  $t$  seconds is given by  $\mathbf{a} = 3(2t - 5)\mathbf{i} + 4\mathbf{j}$ . When  $t = T$ , the magnitude of the horizontal force acting on the ship is 10 N.

Find the possible values of  $T$ .

[4]

- 10 Particles  $P$  and  $Q$ , of masses 3 kg and 5 kg respectively, are attached to the ends of a light inextensible string. The string passes over a smooth fixed pulley. The system is held at rest with the string taut. The hanging parts of the string are vertical and  $P$  and  $Q$  are above a horizontal plane (see diagram).



- (i) Find the tension in the string immediately after the particles are released.

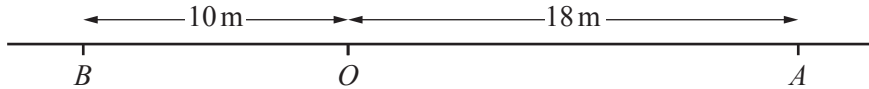
[4]

After descending 2.5 m,  $Q$  strikes the plane and is immediately brought to rest. It is given that  $P$  does not reach the pulley in the subsequent motion.

- (ii) Find the distance travelled by  $P$  between the instant when  $Q$  strikes the plane and the instant when the string becomes taut again.

[4]

11



A particle  $P$  is moving along a straight line with constant acceleration. Initially the particle is at  $O$ . After  $9$  s,  $P$  is at a point  $A$ , where  $OA = 18$  m (see diagram) and the velocity of  $P$  at  $A$  is  $8 \text{ m s}^{-1}$  in the direction  $\overrightarrow{OA}$ .

(i) (a) Show that the initial speed of  $P$  is  $4 \text{ m s}^{-1}$ . [2]

(b) Find the acceleration of  $P$ . [2]

$B$  is a point on the line such that  $OB = 10$  m, as shown in the diagram.

(ii) Show that  $P$  is never at point  $B$ . [4]

A second particle  $Q$  moves along the same straight line, but has variable acceleration. Initially  $Q$  is at  $O$ , and the displacement of  $Q$  from  $O$  at time  $t$  seconds is given by

$$x = at^3 + bt^2 + ct,$$

where  $a$ ,  $b$  and  $c$  are constants.

It is given that

- the velocity and acceleration of  $Q$  at the point  $O$  are the same as those of  $P$  at  $O$ ,
- $Q$  reaches the point  $A$  when  $t = 6$ .

(iii) Find the velocity of  $Q$  at  $A$ . [5]

**END OF QUESTION PAPER**

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